JOURNAL OF BASIC SCIENCE AND ENGINEERING

MODELLING OFABULKFUZZY QUEUESYSTEMWITH TWO PARAMETERS AND THREE PARAMETERSUSING α-CUT METHODWITH TRAPEZOIDAL FUZZY NUMBERS

G. SOMASUNDARA ORI [1] , B.ABIRAMI[(2*)]

1,2 DWARAKA DOSS GOVERDHAN DOSS VAISHNAV COLLEGE,*

AFFILIATED TO UNIVERSITY OF MADRAS, CHENNAI, TAMILNADU, INDIA.

1)soamsundaraori@gmail.com&2) abirami.balagurunathan@gmail.com

(2*) CORRESPONDING AUTHOR

ABSTRACT

In this paper, the membership function of the system performance measure with bulk arrival queues of two and three parameters has been determined through the parametric programming problem approach. This method can be applied to transform a bulk arrival fuzzy queue into a family of bulk arrival crisp queues based on Zadeh's extension concept. The fuzzy bulk arrival queue upon the $FM^{[x]/}FM/S$ model has been solved by a parametric programming problem approach using an α-cut technique with trapezoidal fuzzy numbers. The model is validated using a numerical example using two alternative models: one with a different arrival rate and a uniform service rate, and the other with a different arrival rate and a corresponding service rate.

Key words: Bulk Queuing model, priority queues, parametric programming problem, triangular fuzzy number, trapezoidal fuzzy number.

AMS Mathematics Subject Classification (2010): 90B22; 60K25; 68M20; 03E72

1. INTRODUCTION

Since different clients or consumers are serviced by different types of servers in accordance with defined queue discipline, queueing models are utilized more frequently in real-world situations. The arrival rate and service rate are typically expressed as fact using the language-specific phrases slow, moderate, and quick. However, the best way to communicate slow and moderate rates is to use fuzzy sets, which represent uncertainty in the range [0, 1].

The concept of fuzzy sets was initially developed by Zadeh [9, 10, 11]. The fuzzy queue model isa general method for a queuing system in a fuzzy environment, first introduced by R.J.

LI andE. S. LEE [2]. To see the approach in action, two popular fuzzy queues, represented by M/F/1 and FM/FM/1, are studied.

A. Krishnamoorty and P.V. Ushakumari have studied a Markovian queueing system where units depart separately but batches are accessible for service [1]. Pourdarvish A. & M. Shokry had investigated the $M^{[X]}/M/1$ Queuing System with Fuzzy Parameters [6]. Negi & Lee [5] proposed a novel technique using the α -cut. T.P. Singh and Kusum fuzzified the machine repair queue model [8].

Meenu Mittal [3]et al. have studied a fuzzy queuing model on batch arrival with threshold effect recently. The aim of the research is to represent a bulk queuing system with FCFS discipline in triangular fuzzy numbers using α - cut representation using DSW method. The study's findings were quite favorable.

This study is also an extension of the work by Meenu Mittal et al. [4], since it is believed that the fuzzy environment is trapezoidal in form. The fuzzy set can be arranged into discrete level points for two parameters, three parameters with equal service rates, and different service rates for the corresponding arrival rate by using the α-cut strategy.

The structure of the remaining part of the paper is as follows. We go over the basic definitions, assumptions, and notations of the fuzzy batch queuing system in Section 2. We illustrated a numerical application for two and three parameters with equal service rate and different service rates for the corresponding arrival rate in Sections 3 and 4. Section 6 concludes up with the discussion in Section 5.

2. The fundamental terms and model descriptions

2.1 Preliminaries

In this section we recall some basic definitions.

Definition 2.1. A fuzzy set is a set where the members are allowed to have partialmembership and hence the degree of membership varies from 0 to 1. It is expressed as$\tilde{A}$={(x,$\mu_{\tilde{A}}(X)$)/X∈Z}whereZistheuniverseofdiscourseand$\mu_{\tilde{A}}(X)$isarealnumber, $\mu_{\tilde{A}}(X)$=0 or 1,ie,x is a non-member in $\tilde{A}$ if $\mu_{\tilde{A}}(X)$=0 and X is a member in

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$\tilde{A}$if$\mu_{\tilde{A}}(X)$=1.

Definition 2.2 If a fuzzy set $\tilde{A}$is defined on X , for any α ∈ [0.1]], theα-cuts of thefuzzyset $\tilde{A}$isrepresented by $\tilde{A}_\alpha = \{X/\mu_{\tilde{A}}(X) \geq \alpha, X \in Z\} = \{l_{\tilde{A}}(\alpha), u_{\tilde{A}}(\alpha)\}$,where $l_{\tilde{A}}(\alpha)$and$u_{\tilde{A}}(\alpha)$representthelowerboundandupperboundoftheα-cutof$\tilde{A}$respectively.

Definition2.3. The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degreeα is called the α level set $\tilde{A}_\alpha = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$ where α ∈ [0,1].

Definition2.4. The support of a fuzzy set A is the crisp set such that it is representedassup$\tilde{A}$(X) $= \{x \in X/\mu_{\tilde{A}}(x) > 0\}$. Thus, support of a fuzzy set is the set of all memberswithastrongα-cut,where α=0.

Definition2.5. Trapezoidal fuzzy number

Trapezoidal fuzzy number can be defined as $\tilde{A} = (x_1, x_2, x_3, x_4)$ the membership function of this fuzzy number will be interpreted as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0\,, & x < x_1 \\ \dfrac{x - x_1}{x_2 - x_1}, & x_1 \leq x \leq x_2 \\ 1 & x_2 \leq x \leq x_3 \\ \dfrac{x_4 - x}{x_4 - x_3}, & x_3 \leq x \leq x_4 \\ 0, & x > x_4 \end{cases}$$

Definition2.6. Conversion of Trapezoidal fuzzy number into Interval using α − cut

Let $\tilde{A} = \{a, b, c, d\}$ be the trapezoidal fuzzy number then to fine α −cut of $\tilde{A}$. we first set α equal to the left and right membership function of $\tilde{A}$.

That is $\alpha = \frac{x-a}{b-a}$ and $\alpha = \frac{d-x}{d-c}$.Expressing x in terms of α we have,

$x = \alpha(b - a) + a$ and $x = -\alpha(d - c) + d$.

Therefore we can write the fuzzy interval in terms of α − cut interval:

$\tilde{A}_{\alpha} = [\alpha(b-a)+a \ , \ -\alpha(d-c)+d]$.

Definition2.7. ($FM^{[X]/}FM/1$): (FCFS/∞/∞) Model

Single server fuzzy queue infinite calling source and first come first served discipline. In technically ($FM^{[X]/}FM/1$): (FCFS/∞/∞). All the customers in the waiting line will be served by single server. The arrival time and the service time follow poison and exponential distribution.

Definition2.8. The interval calculation

LetI_1andI_2betwointervalnumbersdefinedbyorderedofreal numberswithlowerandupperbounds.I_1=[a,b],a≤b&I_2=[c,d], c ≤d. Defineageneralarithmeticpropertywiththesymbol.

* = [+, -, ×, ÷] symbolically the operation $I_1 * I_2$ = [a, b] * [c, d] represents another interval.The interval calculation depends on the magnitudes and signs of the elements a, b, c, d.

$[a,b]+[c,d]=[a+c,b+d]$
$[a,b]-[c,d]=[a-d,b-c]$

$[a,b]\times[c,d]=[\min(ac,ad,bc,bd),max(ac,ad,bc,bd)]$

$[a,b]\div[c,d]=[a,b]\times\left[\frac{1}{d},\frac{1}{c}\right],provided 0\notin[c,d]$.

Definition2.9.Technique to apply$a$-cut:

Letusconsideraclassicalsingleserverqueueingmodelinfinitecallingsourceandfir stcomefirstserveddiscipline.

TheinterarrivaltimeAandtheservicetimeSaredescribedbythefollowingfuzzysets

A={(a,$\mu_{\tilde{A}}$(a)):a∈X}, S={(s,$\mu_{\tilde{S}}$(s)):s∈Y}

HereXistheclassicalsetoftheinterarrivaltimeandYistheclassicalsetoftheservicetime.

$\tilde{\mu}_A$(a)is themembershipfunctionof theinterarrivaltime.
$\tilde{\mu}_S$(s)is themembershipfunctionof theservice time.

Theα–cutofinterarrivaltimeandservicetimearerepresentedas

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

A(α)={aϵX: $\tilde{\mu}_A$(a)≥α}, S(α)={s ϵY: $\tilde{\mu}_S$(s)≥α}

2.2 Model description:

Bearing the consideration of a single service channel with a Poisson input of batch size b, fuzzified exponential inter-arrival service model of a queue system with infinite capacity, and FCFS service discipline, the model is of type $M^{[b]/}$M/1/∞/FCFS. Let 'b' be the batchsize b Poisson Process arrival rate and let C_b be the assigned probability. The number of customers in any arrival is determined by the random variable b,

where $\boldsymbol{C_b} = \frac{\lambda_b}{\lambda}$and is the average arrival rate for all batches of size b, i.e.,$\boldsymbol{\lambda} = \sum_{i=1}^{\infty} \boldsymbol{\lambda_i}$.

We also define Pn (t), the probability that there are n units in the system at any time during t, based on Meenu Mittal, T.P. Singh, and Deepak Gupta [4]. It is feasible to derive the model's differential difference equations using general Birth Death arguments.

$P_n^{'}(t) = -(\lambda + \mu)P_n(t) + \mu P_{n+1}(t)$+λ$\sum_{i=1}^{\infty}. P_{n+1}(t)c_k$, $n \geq 1$
$P_0^{'}(t) = -\lambda P_0(t) + \mu P_1(t)$, n = 0 …………(1)

When the system's behavior begins to depend on the course of time, the steady state condition has been reached. When$t$ → ∞thesteadystateequationsare

0=-(λ+μ)P_n+μP_{n+1}+$\lambda\sum_{k=1}^{n} P_{n-k}C_k$n≥1

0=-λP_0+μP_1n=0 ………(2)

Thevariousperformancemeasuresofthismodelarederivedby Meenu Mittal, T.P. Singh, and Deepak Gupta [3].

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

3.Trapezoidal fuzzy number with service rate μ

3.1 . Trapezoidal fuzzy number for Two Parameters

1. ThesteadystateProbability$P_n$isgivenby

$P_n=(1-\rho)\{\alpha+(1-\alpha)\rho\}^{n-1}(1-\alpha)\rho$; $n\geq 0$ where$\rho = \frac{\lambda}{\mu(1-\lambda)}$**……(3)**

2. ExpectedbatchsizeE(x)fortwoparametersλ_1&$\lambda_2$isgivenby

$E(X) = \frac{(\lambda_1+2\lambda_2)}{\lambda}$**…………(4)**

3. ExpectedqueuelengthL$_q$fortwoparametersλ_1&$\lambda_2$isgivenby

$L_q = \left[\frac{\rho_1+3\rho_2}{1-\rho_1-2\rho_2}\right]$ where $\rho = \rho_1 + 2\rho_2 < 1$; $\rho_1 = \frac{\lambda_1}{\mu}$, $\rho_2 = \frac{\lambda_2}{\mu}$ **…..…….(5)**

4. Meanqueuelengthinfuzzyparametersisgivenby

$$\bar{L}_q(\alpha) = \left[\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}}, \quad \frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}}\right] \text{……………………(6)}$$

5. ByusingLittle'sformulae,wemayfindothereffectivemeasuresinfuzzyparameters

(a) waitingtimeinqueue:

$$\bar{W}_q(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}}\right); \ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}}\right)\right] \text{ where } \bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i \text{………(7)}$$

(b)waitingtimeinthe system

$$\bar{W}_s(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}}\right) + \frac{1}{\bar{\mu}_U}; \ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}}\right) + \frac{1}{\bar{\mu}_L}\right] \text{……………………(8)}$$

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

where $\bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i$ and $\bar{\mu} = \sum_{i=1}^{\infty} \bar{\mu}_i$

3.2Numerical illustrations fortrapezoidal fuzzy number with Two Parameters

Consider both arrival and service rates as trapezoidal fuzzy number as

$\bar{\lambda}_1 = (2,4,6,8)$, $\bar{\lambda}_2 = (3,5,7,9)$, $\bar{\mu} = (38,40,42,44)$ such that $\bar{\rho} = \frac{(\bar{\lambda}_1 + 2\bar{\lambda}_2)}{\bar{\mu}} < 1$

Applying $\alpha - cut$, the interval of confidence at possibility level αas(by definition 2.6)

$$\bar{\lambda}_1 = (2\alpha + 2, 8 - 2\alpha),$$

$$\bar{\lambda}_2 = (2\alpha + 3, 9 - 2\alpha),$$

$$\bar{\mu} = (2\alpha + 38, 44 - 2\alpha),$$

$\bar{\lambda} = (4\alpha + 5, 17 - 4\alpha)$,Where $\bar{\lambda} = (\bar{\lambda}_1 + \bar{\lambda}_2)$

$$\bar{\rho}_1 = \left(\frac{2\alpha+2}{44-2\alpha} \;,\; \frac{8-2\alpha}{2\alpha+38}\right),$$

$$\bar{\rho}_2 = \left(\frac{2\alpha+3}{44-2\alpha} \;,\; \frac{9-2\alpha}{2\alpha+38}\right)$$

Using, equations (6), (7) and (8) respectively,

$$\bar{L}_q(\alpha) = \left(\frac{8\alpha+11}{36-8\alpha} \;,\; \frac{35-8\alpha}{8\alpha+12}\right),$$

$$\bar{W}_q(\alpha) = \left[\left(\frac{1}{17-4\alpha}\left(\frac{8\alpha+11}{36-8\alpha}\right)\right) ,\left(\frac{1}{4\alpha+5}\left(\frac{35-8\alpha}{8\alpha+12}\right)\right)\right],$$

$$\bar{W}_s(\alpha) = \left[\left(\frac{1}{17-4\alpha}\left(\frac{8\alpha+11}{36-8\alpha}\right) + \frac{1}{44-2\alpha}\right) ,\left(\frac{1}{4\alpha+5}\left(\frac{35-8\alpha}{8\alpha+12}\right) + \frac{1}{2\alpha+38}\right)\right] \text{……………………………..}\mathbf{(9)}$$

$Table$: 1 $The\ \alpha - cuts\ of\ \bar{L}_q, \bar{W}_q(\alpha)\ and \bar{W}_s\ (\alpha)$ at α values for two parameters.

S.No	α	$\bar{L}_q$ [$\bar{L}_q$Lower, $\bar{L}_q$	$\bar{W}_q(\alpha)$ [$\bar{W}_q$ $Lower$,	$\bar{W}_s(\alpha)$ [$\bar{W}_s$ $Lower$,

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

		Upper]	$\overline{W}_q$ $Upper]$	$\overline{W}_s$ $Upper]$
1	0	[0.3056, 2.9167]	[0.0180, 0.5833]	[0.0407, 0.6096]
2	0.1	[0.3352, 2.6719]	[0.0202, 0.4948]	[0.0430, 0.5210]
3	0.2	[0.3663, 2.4559]	[0.0226, 0.4234]	[0.0455, 0.4495]
4	0.3	[0.3988, 2.2639]	[0.0252, 0.3651]	[0.0483, 0.3911]
5	0.4	[0.4329, 2.09210]	[0.0281, 0.3170]	[0.0513, 0.3428]
6	0.5	[0.4688, 1.9375]	[0.0313, 0.2768]	[0.0545, 0,3024]
7	0.6	[0.5064, 1.7976]	[0.0347, 0.2429]	[0.0581, 0.2684]
8	0.7	[0.5461, 1.6705]	[0.0384, 0.2142]	[0.0619, 0.2395]
9	0.8	[0.5878, 1.5543]	[0.0426, 0.1896]	[0.0662, 0.2148]
10	0.9	[0.6319, 1.4479]	[0.0472, 0.1683]	[0.0709, 0.1935]
11	1	[0.6786, 1.3500]	[0.0522, 0.1500]	[0.0760, 0.1750]

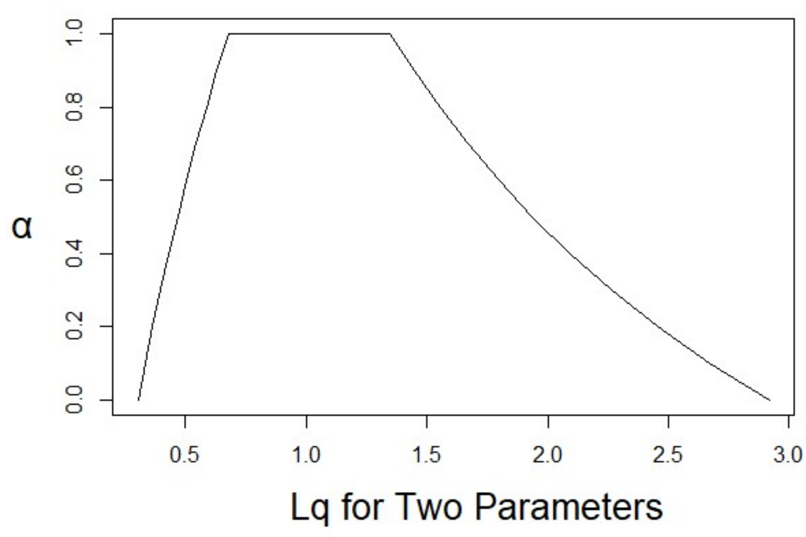

Fig. 1 The membership function of the Trapezoidal Expected queue length Lq of two parameters

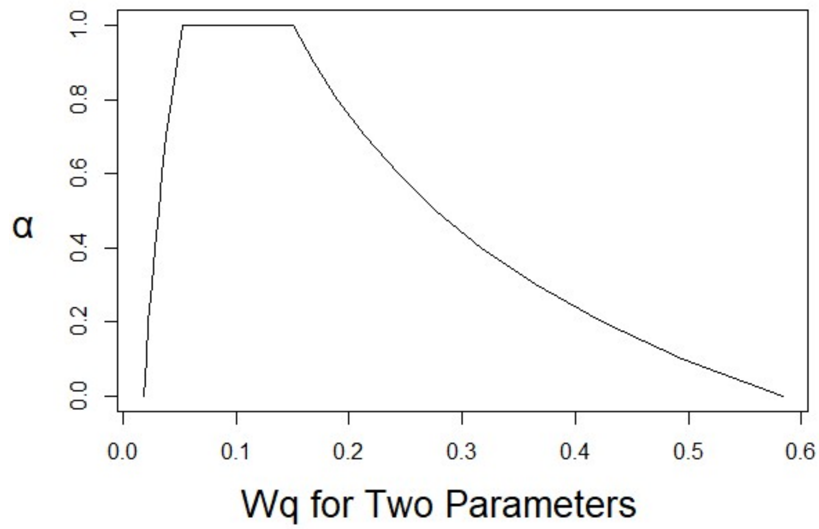

Fig. 2 The membership function of the Trapezoidal Expected waiting time in the queue Wqof Two Parameters

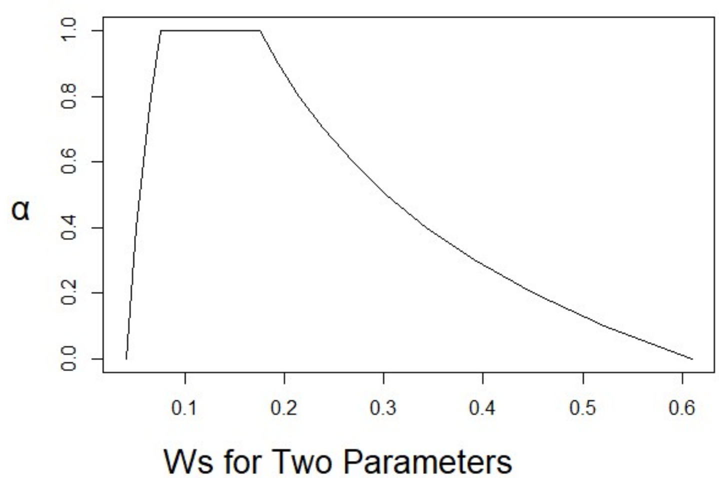

Fig.3 The membership function of the Trapezoidal Expected waiting time in the system Ws of Two Parameters

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

3.3 Numerical illustrations fortrapezoidal fuzzy number with Three Parameters

1. ExpectedbatchsizeE(x)forthreeparametersλ_1,λ_2&$\lambda_3$isgivenby

$E(X) = \frac{(\lambda_,+2\lambda_2+3\lambda_3)}{\lambda}$**………………(10)**

2. Expectedqueuelengthforthree parametersλ_1,λ_2&$\lambda_3$is givenby

$L_q = \left[\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right]$ where $\rho = \rho_1 + 2\rho_2 + 3\rho_3 < 1;\ \rho_1 = \frac{\lambda_1}{\mu}\ ,\ \rho_2 = \frac{\lambda_2}{\mu}\ ,\ \rho_3 = \frac{\lambda_3}{\mu}$**……(11)**

3. Expectedwaiting time in the queueforthree parametersλ_1,λ_2&$\lambda_3$is givenby

$W_q = \left[\left(\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right)\left(\frac{1}{\lambda}\right)\right]$ where $\rho_1 = \frac{\lambda_1}{\mu}\ ,\ \rho_2 = \frac{\lambda_2}{\mu}\ ,\ \rho_3 = \frac{\lambda_3}{\mu}$and $\bar{\lambda} = \sum_{i=1}^{\infty}\bar{\lambda}_i$**………(12)**

4. Expectedwaiting time in the queueforthree parametersλ_1,λ_2&$\lambda_3$is givenby

$W_s = \left[\left(\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right)\left(\frac{1}{\lambda}\right) + \frac{1}{\mu}\right]$ where $\rho_1 = \frac{\lambda_1}{\mu}\ ,\ \rho_2 = \frac{\lambda_2}{\mu}\ ,\ \rho_3 = \frac{\lambda_3}{\mu}\ ,\ \bar{\lambda} = \sum_{i=1}^{\infty}\bar{\lambda}_i$ and $\bar{\mu} = \sum_{i=1}^{\infty}\bar{\mu}_i$

……….(13)

5. Meanqueuelengthinfuzzyparametersisgivenby

$\bar{L}_q(\alpha) = \left[\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}+6\bar{\rho}_{(3,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}-3\bar{\rho}_{(3,L)}},\ \ \frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}+6\bar{\rho}_{(3,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}-3\bar{\rho}_{(3,U)}}\right]$**………………(14)**

6. ByusingLittle'sformulae,wemayfindothereffectivemeasuresinfuzzyparameters

(a) waitingtimeinqueue:

$\bar{W}_q(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}+6\bar{\rho}_{(3,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}-3\bar{\rho}_{(3,L)}}\right);\ \ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}+6\bar{\rho}_{(3,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}-3\bar{\rho}_{(3,U)}}\right)\right]$ where $\bar{\lambda} = \sum_{i=1}^{\infty}\bar{\lambda}_i$

………………………………(15)

(b) waitingtimeinthe system

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$$\bar{W}_s(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)} + 3\bar{\rho}_{(2,L)} + 6\bar{\rho}_{(3,L)}}{1 - \bar{\rho}_{(1,L)} - 2\bar{\rho}_{(2,L)} - 3\bar{\rho}_{(3,L)}}\right) + \frac{1}{\bar{\mu}_U};\ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)} + 3\bar{\rho}_{(2,U)} + 6\bar{\rho}_{(3,U)}}{1 - \bar{\rho}_{(1,U)} - 2\bar{\rho}_{(2,U)} - 3\bar{\rho}_{(3,U)}}\right) + \frac{1}{\bar{\mu}_L}\right]$$

………………………………(16)

where $\bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i$and $\bar{\mu} = \sum_{i=1}^{\infty} \bar{\mu}_i$

3.4 Numerical illustrations for trapezoidal fuzzy number with Three Parameters

Consider both arrival and service rates as trapezoidal fuzzy number as

$\bar{\lambda}_1 = (2,5,8,11)$, $\bar{\lambda}_2 = (3,6,9,12), \bar{\lambda}_3 = (4,7,10,13)$, $\bar{\mu} = (125,128,131,134)$

such that $\bar{\rho} = \frac{(\bar{\lambda}_1 + 2\bar{\lambda}_2 + 3\bar{\lambda}_3)}{\bar{\mu}} < 1$

Applying $\alpha - cut$, the interval of confidence at possibility level αas(by definition 2.6)

$\bar{\lambda}_1 = (3\alpha + 2, 11 - 3\alpha)$, $\bar{\lambda}_2 = (3\alpha + 3, 12 - 3\alpha)$, $\bar{\lambda}_3 = (3\alpha + 4, 13 - 3\alpha)$,

$\bar{\mu} = (3\alpha + 125, 134 - 3\alpha), \bar{\lambda} = (9\alpha + 9,\ 36 - 9\alpha)$ Where $\bar{\lambda} = (\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{\lambda}_3)$

$\bar{\rho}_1 = \left(\frac{3\alpha+2}{134-3}\ ,\ \frac{11-3\alpha}{3\alpha+125}\right)$,

$\bar{\rho}_2 = \left(\frac{3\alpha+3}{134-3\alpha}\ ,\ \frac{12-3\alpha}{3\alpha+12}\right)$,

$$\bar{\rho}_3 = \left(\frac{3\alpha + 4}{134 - 3\alpha}\ ,\ \frac{13 - 3\alpha}{3\alpha + 125}\right)$$

Using, equations (14), (15) and (16) respectively,

$\bar{L}_q(\alpha) = \left(\frac{30\alpha+3}{114-2}\ ,\ \frac{125-30\alpha}{51+21\alpha}\right)$,

$\bar{W}_q(\alpha) = \left[\left(\frac{1}{36-9\alpha}\left(\frac{30\alpha+3}{114-21\alpha}\right)\right)\ ,\left(\frac{1}{9\alpha+9}\left(\frac{125-30\alpha}{51+21\alpha}\right)\right)\right]$,

$\bar{W}_s(\alpha) = \left[\left(\frac{1}{36-9\alpha}\left(\frac{30\alpha+35}{114-21\alpha}\right) + \frac{1}{134-3}\right)\ ,\left(\frac{1}{9\alpha+9}\left(\frac{125-30\alpha}{51+21\alpha}\right) + \frac{1}{3\alpha+12}\right)\right]$**…………..(17)**

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$\boldsymbol{Table}$: $\mathbf{2}$ $The\ \alpha - cuts\ of\ \bar{L}_q, \bar{W}_q(\alpha)\ and \bar{W}_s\ (\alpha)$ at α values for three parameters.

S.No	α	$\bar{L}_q$ [$\bar{L}_q$Lower, $\bar{L}_q$ Upper]	$\bar{W}_q(\alpha)$ [$\bar{W}_q\ Lower$, $\bar{W}_q\ Upper$]	$\bar{W}_s(\alpha)$ [$\bar{W}_s\ Lower$, $\bar{W}_s\ Upper$]
1	0	[0.3070, 2.4310]	[0.0085, 0.2723]	[0.0160, 0.2803]
2	0.1	[0.3396, 2.2976]	[0.0097, 0.2321]	[0.0172, 0.2401]
3	0.2	[0.3734, 2.1558]	[0.0109, 0.1996]	[0.0184, 0.2076]
4	0.3	[0.4085, 2.0244]	[0.0123, 0.1730]	[0.0198, 0.1810]
5	0.4	[0.4451, 1.9024]	[0.0137, 0.1510]	[0.0213, 0.1589]
6	0.5	[0.4831, 1.7886]	[0.0153, 0.1325]	[0.0229, 0.1404]
7	0.6	[0.5227, 1.6824]	[0.0171, 0.1168]	[0.0247, 0.1247]
8	0.7	[0.5639, 1.5830]	[0.0190, 0.1035]	[0.0266, 0.1113]
9	0.8	[0.6070, 1.4897]	[0.0211, 0.0920]	[0.0287, 0.0998]
10	0.9	[0.6519, 1.4020]	[0.0234, 0.0820]	[0.0310, 0.0898]
11	1	[0.6989, 1.3194]	[0.0259, 0.0733]	[0.0335, 0.0811]

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

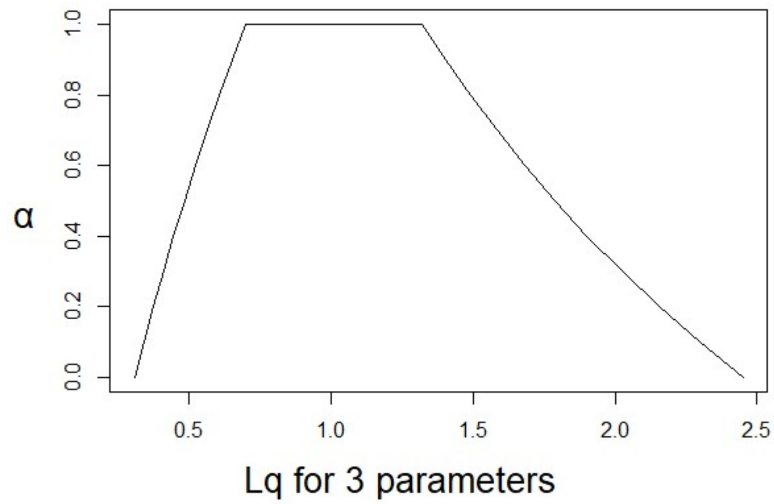

Fig. 4 The membership function of the Trapezoidal Expected queue length Lq of three parameters

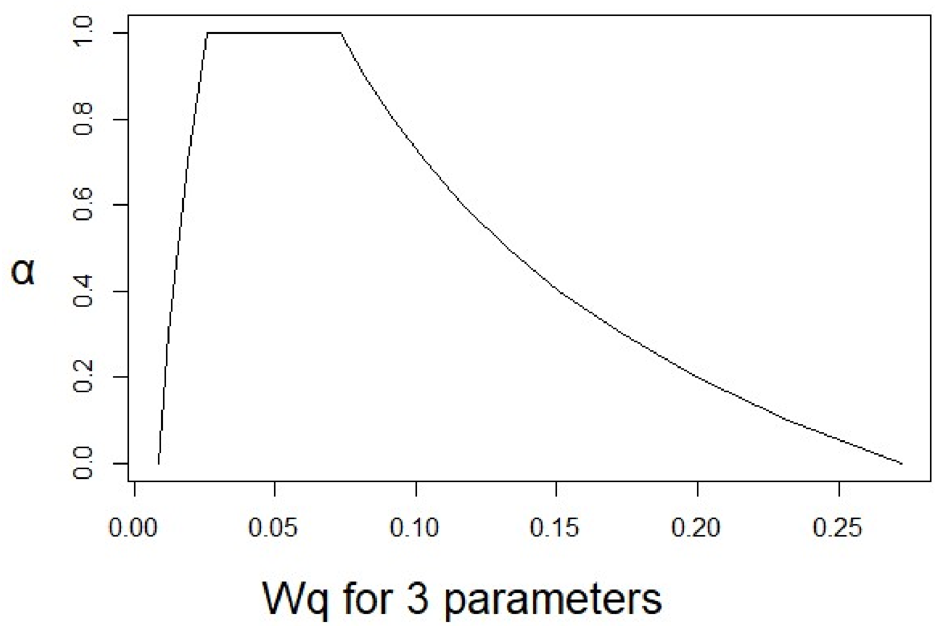

Fig. 5 The membership function of the Trapezoidal Expected waiting time in the queue Wqof three Parameters

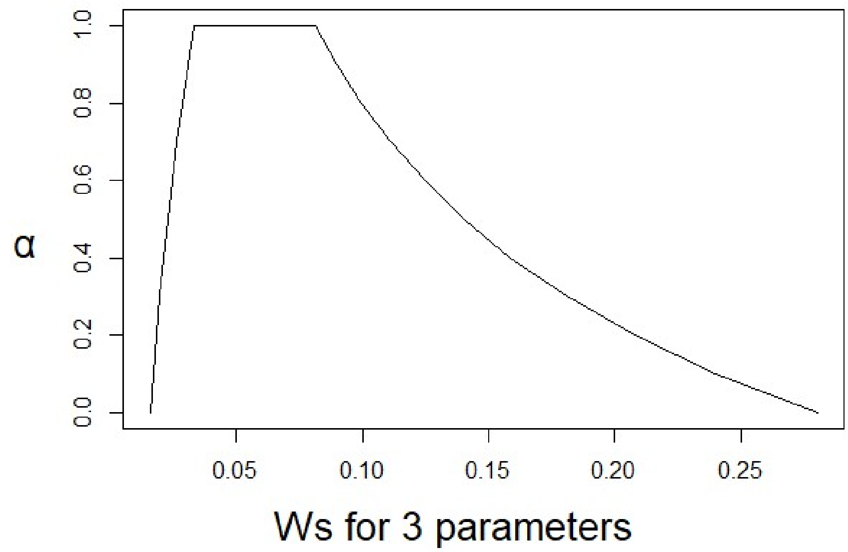

Fig.6 The membership function of the Trapezoidal Expected waiting time in the system Ws of three Parameters

4.Trapezoidal fuzzy number with different service rates to the corresponding arrival rates

Usually, service rates are consistent upon batch arrival, but in many real-world scenarios, service times will vary. In that case, some services take less time than others, and vice versa. Therefore, using the above mentioned model, we explore different service times for batch arrival and evaluate thewith performance measures.

4.1. Trapezoidal fuzzy number for Two Parameters with two service rates to the corresponding arrival rates

For Two Parameters in trapezoidal fuzzy number with different service rate

Consider both arrival and service rates as trapezoidal fuzzy number as

$$\bar{\lambda}_1 = (2,4,6,8), \qquad \bar{\lambda}_2 = (3,5,7,9), \quad \bar{\mu}_1 = (20,22,24,26), \; \bar{\mu}_2 = (40,42,44,46)$$

Applying $\alpha - cut$, the interval of confidence at possibility level α as (by Definition 2.6)

$$\bar{\lambda}_1 = (2\alpha + 2, 8 - 2\alpha),$$

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$$\bar{\lambda}_2 = (2\alpha + 3, 9 - 2\alpha),$$

$$\bar{\mu}_1 = (2\alpha + 20, 26 - 2\alpha),$$

$$\bar{\mu}_2 = (2\alpha + 40, 46 - 2\alpha),$$

$\bar{\lambda} = (4\alpha + 5, 17 - 4\alpha)$and

$\bar{\mu} = (4\alpha + 60, 72 - 4\alpha)$Where $\bar{\lambda} = (\bar{\lambda}_1 + \bar{\lambda}_2)$ and $\bar{\mu} = (\bar{\mu}_1 + \bar{\mu}_2)$

$\bar{\rho}_1 = \left(\frac{2\alpha+2}{26-2\alpha} \ , \ \frac{8-2\alpha}{2\alpha+20}\right),$

$$\bar{\rho}_2 = \left(\frac{2\alpha + 3}{46 - 2\alpha} \ , \ \frac{9 - 2\alpha}{2\alpha + 40}\right)$$

(1) Meanqueuelengthinfuzzyparametersisgivenby

$$\bar{L}_q(\alpha) = \left[\frac{\bar{\rho}_{(1,L)} + 3\bar{\rho}_{(2,L)}}{1 - \bar{\rho}_{(1,L)} - 2\bar{\rho}_{(2,L)}}, \quad \frac{\bar{\rho}_{(1,U)} + 3\bar{\rho}_{(2,U)}}{1 - \bar{\rho}_{(1,U)} - 2\bar{\rho}_{(2,U)}}\right]$$

$\bar{L}_q(\alpha) = \left(\frac{-16\alpha^2+226\alpha+326}{16\alpha^2-324\alpha+948} \ , \ \frac{-16\alpha^2-130\alpha+860}{16\alpha^2+288\alpha+120}\right)$**………..(18)**

(2) waitingtimeinqueue:

$$\bar{W}_q(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)} + 3\bar{\rho}_{(2,L)}}{1 - \bar{\rho}_{(1,L)} - 2\bar{\rho}_{(2,L)}}\right); \ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)} + 3\bar{\rho}_{(2,U)}}{1 - \bar{\rho}_{(1,U)} - 2\bar{\rho}_{(2,U)}}\right)\right]$$

$\bar{W}_q(\alpha) = \left[\left(\frac{1}{17-4\alpha}\left(\frac{-16\alpha^2+226\alpha+326}{16\alpha^2-324\alpha+948}\right)\right) \ ,\left(\frac{1}{4\alpha+5}\left(\frac{-16\alpha^2-130\alpha+860}{16\alpha^2+288\alpha+120}\right)\right)\right]$**…….(19)**

(3) waitingtimeinthe system

$$\bar{W}_s(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)} + 3\bar{\rho}_{(2,L)}}{1 - \bar{\rho}_{(1,L)} - 2\bar{\rho}_{(2,L)}}\right) + \frac{1}{\bar{\mu}_U}; \ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)} + 3\bar{\rho}_{(2,U)}}{1 - \bar{\rho}_{(1,U)} - 2\bar{\rho}_{(2,U)}}\right) + \frac{1}{\bar{\mu}_L}\right]$$

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$$\bar{W}_s(\alpha) = \left[\left(\frac{1}{17-4\alpha}\left(\frac{-16\alpha^2+226\alpha+326}{16\alpha^2-324\alpha+948}\right)+\frac{1}{72-4\alpha}\right), \left(\frac{1}{4\alpha+5}\left(\frac{-16\alpha^2-130\alpha+860}{16\alpha^2+288\alpha+120}\right)+\frac{1}{4\alpha+60}\right)\right]$$

…………….(20)

$\boldsymbol{Table}$: **3** $The\ \alpha - cuts\ of\ \bar{L}_q, \bar{W}_q(\alpha)\ and \bar{W}_s\ (\alpha)$ at α values for two parameters for different service rate

S.No	α	$\bar{L}_q$ [$\bar{L}_q$Lower, $\bar{L}_q$ Upper]	$\bar{W}_q(\alpha)$ [$\bar{W}_q$ $Lower$, $\bar{W}_q$ $Upper$]	$\bar{W}_s(\alpha)$ [$\bar{W}_s$ $Lower$, $\bar{W}_s$ $Upper$]
1	0	[0.3439, 7.1667]	[0.0202, 1.4333]	[0.0341, 1.4500]
2	0.1	[0.3805, 5.9236]	[0.0229, 1.0970]	[0.0369, 1.1135]
3	0.2	[0.4193, 5.0130]	[0.0259, 0.8643]	[0.0399, 0.8808]
4	0.3	[0.4604, 4.3171]	[0.0291, 0.6963]	[0.0433, 0.7126]
5	0.4	[0.5041, 3.7680]	[0.0327, 0.5709]	[0.0469, 0.5871]
6	0.5	[0.5506, 3.3235]	[0.0367, 0.4748]	[0.0510, 0.4909]
7	0.6	[0.6003, 2.9564]	[0.0411, 0.3995]	[0.0555, 0.4155]
8	0.7	[0.6534, 2.9564]	[0.0460, 0.3395]	[0.0605, 0.3554]
9	0.8	[0.7103, 2.3854]	[0.0515, 0.2909]	[0.0660, 0.3067]
10	0.9	[0.7715, 2.1589]	[0.0576, 0.2510]	[0.0722, 0.2668]
11	1	[0.8375, 1.9615]	[0.0644, 0.2179]	[0.0792, 0.2336]

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

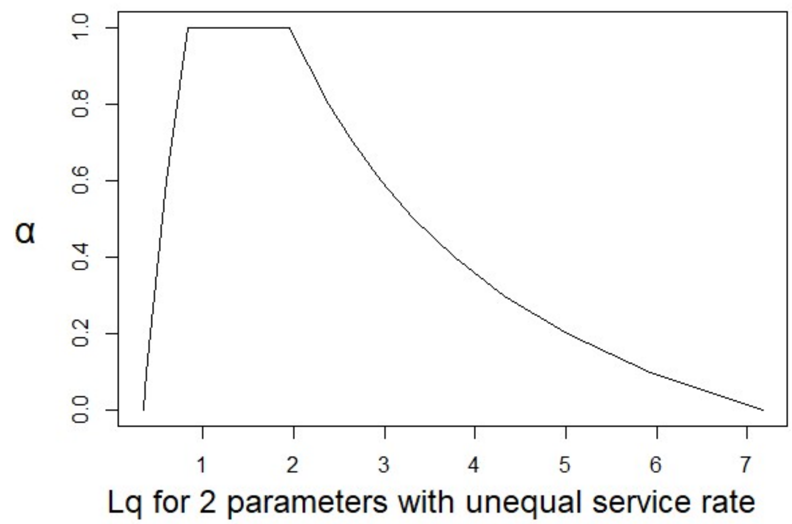

Fig. 7 The membership function of the Trapezoidal Expected queue length Lq of two parameters with corresponding arrival and service rate.

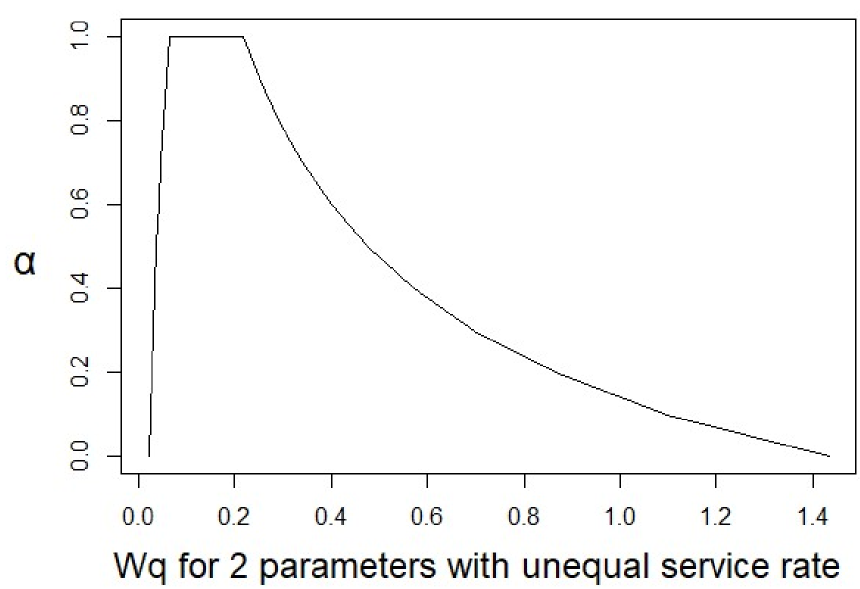

Fig. 8 The membership function of the TrapezoidalExpected waiting time in the

queue Wqof Two Parameters with corresponding arrival and service rates

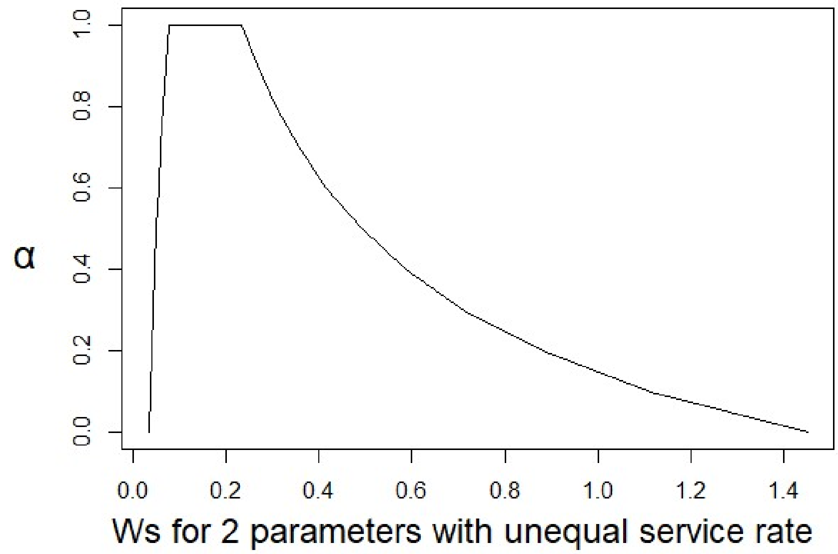

Fig.9 The membership function of the TrapezoidalExpected waiting time in the system Ws of Two Parameters with corresponding arrival and service rate

4.1. Trapezoidal fuzzy number for Three Parameters with corresponding arrival and service rate

Consider both arrival and service rates as trapezoidal fuzzy number as

$$\bar{\lambda}_1 = (1,2,3,4), \quad \bar{\lambda}_2 = (2,3,4,5), \quad \bar{\lambda}_3 = (3,4,5,6)\ ,$$

$$\overline{\mu}_1 = (2,3,4,5), \quad \bar{\mu}_2 = (3,4,5,6), \bar{\mu}_3 = (4,5,6,7)$$

Applying $\alpha - cut$, the interval of confidence at possibility level α as (by Definition 2.6)

$$\bar{\lambda}_1 = (\alpha + 1, \quad 4 - \alpha), \quad \bar{\lambda}_2 = (\alpha + 2, \quad 5 - \alpha), \quad \bar{\lambda}_3 = (\alpha + 3, \quad 6 - \alpha),$$

$$\bar{\mu}_1 = (\alpha + 2, \quad 5 - \alpha), \quad \bar{\mu}_2 = (\alpha + 3, \quad 6 - \alpha), \bar{\mu}_3 = (\alpha + 4, \quad 7 - \alpha)$$

$$\bar{\mu} = (3\alpha + 9, 18 - 3\alpha),$$

$\bar{\lambda} = (3\alpha + 6,\ \ 15 - 3\alpha)$ Where $\bar{\lambda} = \left(\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{\lambda}_3\right)$, and $\bar{\mu} = (\bar{\mu}_1 + \bar{\mu}_2 + \bar{\mu}_3)$

$\bar{\rho}_1 = \left(\frac{\alpha+1}{5-\alpha}\ ,\ \ \frac{4-\alpha}{\alpha+2}\right),$

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$\bar{\rho}_2 = \left(\frac{\alpha+2}{6-\alpha}\ ,\ \frac{5-\alpha}{\alpha+3}\right),$

$$\bar{\rho}_3 = \left(\frac{\alpha+3}{7-\alpha}\ ,\ \frac{6-\alpha}{\alpha+4}\right)$$

(1) ExpectedbatchsizeE(x)forthreeparametersλ_1,λ_2&$\lambda_3$isgivenby

$E(X) = \frac{(\lambda_{,}+2\lambda_2+3\lambda_3)}{\lambda}$**............(21)**

(2) Expectedqueuelengthforthree parametersλ_1,λ_2&$\lambda_3$is givenby

$L_q = \left[\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right]$ where $\rho = \rho_1 + 2\rho_2 + 3\rho_3 < 1;\ \ \rho_1 = \frac{\lambda_1}{\mu_1}\ \ ,\ \rho_2 = \frac{\lambda_2}{\mu_2}\ \ ,\ \rho_3 = \frac{\lambda_3}{\mu_3}$

$$\bar{L}_q(\alpha) = \left[\frac{\bar{\rho}_{(1,L)} + 3\bar{\rho}_{(2,L)} + 6\bar{\rho}_{(3,L)}}{1-\bar{\rho}_{(1,L)} - 2\bar{\rho}_{(2,L)} - 3\bar{\rho}_{(3,L)}},\ \ \frac{\bar{\rho}_{(1,U)} + 3\bar{\rho}_{(2,U)} + 6\bar{\rho}_{(3,U)}}{1-\bar{\rho}_{(1,U)} - 2\bar{\rho}_{(2,U)} - 3\bar{\rho}_{(3,U)}}\right]$$

$\bar{L}_q(\alpha) = \left(\frac{10\ \ ^3-90\alpha^2+44\alpha+792}{-7\alpha^3+74\alpha^2-149\alpha-242}\ ,\ \frac{-10\alpha^3+226\alpha+384}{7\alpha^3+11\alpha^2-106\alpha-212}\right)$**..............(22)**

(3) Expectedwaiting time in the queueforthree parametersλ_1,λ_2&$\lambda_3$is givenby

$W_q = \left[\left(\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right).\left(\frac{1}{\lambda}\right)\right]$ where $\rho_1 = \frac{\lambda_1}{\mu_1}\ \ ,\ \rho_2 = \frac{\lambda_2}{\mu_2}\ \ ,\ \rho_3 = \frac{\lambda_3}{\mu_3}$ and $\bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i$

$\bar{W}_q(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}+6\bar{\rho}_{(3,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}-3\bar{\rho}_{(3,L)}}\right);\ \ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}+6\bar{\rho}_{(3,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}-3\bar{\rho}_{(3,U)}}\right)\right]$ where $\bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i$

$$\bar{W}_q(\alpha) = \left[\left(\frac{1}{15-3\alpha}\left(\frac{10\alpha^3-90\alpha^2+44\alpha+792}{-7\alpha^3+74\alpha^2-149\alpha-242}\right)\right)\ ,\left(\frac{1}{3\alpha+6}\left(\frac{-10\alpha^3+226\alpha+384}{7\alpha^3+11\alpha^2-106\alpha-212}\right)\right)\right]$$

........................(23)

(4) Expectedwaiting time in the systemforthree parametersλ_1,λ_2&$\lambda_3$is givenby

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$W_s = \left[\left(\frac{\rho_1+3\rho_2+6\rho_3}{1-\rho_1-2\rho_2-3\rho_3}\right).\left(\frac{1}{\bar{\lambda}}\right)+\frac{1}{\bar{\mu}}\right]$ where $\rho_1 = \frac{\lambda_1}{\mu_1}$, $\rho_2 = \frac{\lambda_2}{\mu_2}$, $\rho_3 = \frac{\lambda_3}{\mu_3}$, $\bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i$ and $\bar{\mu} = \sum_{i=1}^{\infty} \bar{\mu}_i$

$$\bar{W}_s(\alpha) = \left[\frac{1}{\bar{\lambda}_U}\left(\frac{\bar{\rho}_{(1,L)}+3\bar{\rho}_{(2,L)}+6\bar{\rho}_{(3,L)}}{1-\bar{\rho}_{(1,L)}-2\bar{\rho}_{(2,L)}-3\bar{\rho}_{(3,L)}}\right)+\left(\frac{1}{\bar{\mu}_U}\right);\ \frac{1}{\bar{\lambda}_L}\left(\frac{\bar{\rho}_{(1,U)}+3\bar{\rho}_{(2,U)}+6\bar{\rho}_{(3,U)}}{1-\bar{\rho}_{(1,U)}-2\bar{\rho}_{(2,U)}-3\bar{\rho}_{(3,U)}}\right)+\left(\frac{1}{\bar{\mu}_L}\right)\right]$$

where $\bar{\lambda} = \sum_{i=1}^{\infty} \bar{\lambda}_i$

$$\bar{W}_s(\alpha) = \left[\left(\frac{1}{15-3\alpha}\left(\frac{10\alpha^3-90\alpha^2+44\alpha+792}{-7\alpha^3+74\alpha^2-149\alpha-242}\right)\right) + \left(\frac{1}{18-3\alpha}\right),\left(\frac{1}{3\alpha+6}\left(\frac{-10\alpha^3+226\alpha+384}{7\alpha^3+11\alpha^2-106\alpha-212}\right)\right)+\left(\frac{1}{3\alpha+9}\right)\right]$$

…………………………..(24)

***Table*: 4** *The $\alpha -$ cuts of $\bar{L}_q, \bar{W}_q(\alpha)$ and $\bar{W}_s(\alpha)$* at α values for 3 parameters for corresponding arrival and service rate

S.No	α	$\bar{L}_q$ [$\bar{L}_q$Lower, $\bar{L}_q$ Upper]	$\bar{W}_q(\alpha)$ [$\bar{W}_q$ Lower, $\bar{W}_q$ Upper]	$\bar{W}_s(\alpha)$ [$\bar{W}_s$ Lower, $\bar{W}_s$ Upper]
1	0	[0.4655, 5.3537]	[0.0310, 0.8923]	[0.0367, 0.8982]
2	0.1	[0.5024, 4.6743]	[0.0342, 0.7420]	[0.0398, 0.7479]
3	0.2	[0.5411, 4.1300]	[0.0376, 0.6258]	[0.0432, 0.6317]
4	0.3	[0.5820, 3.6839]	[0.0413, 0.5339]	[0.0470, 0.5398]
5	0.4	[0.6252, 3.3116]	[0.0453, 0.4599]	[0.0510, 0.4659]
6	0.5	[0.6710, 2.9961]	[0.0497, 0.3995]	[0.0554, 0.4054]
7	0.6	[0.7194, 2.7252]	[0.0545, 0.3494]	[0.0602, 0.3553]

8	0.7	[0.7709, 2.4901]	[0.0598, 0.3074]	[0.0655, 0.3133]
9	0.8	[0.8256, 2.2841]	[0.0655, 0.2719]	[0.0713, 0.2778]
10	0.9	[0.8840, 2.1019]	[0.0719, 0.2416]	[0.0776, 0.2475]
11	1	[0.9464, 1.9398]	[0.0789, 0.2155]	[0.0846, 0.2214]

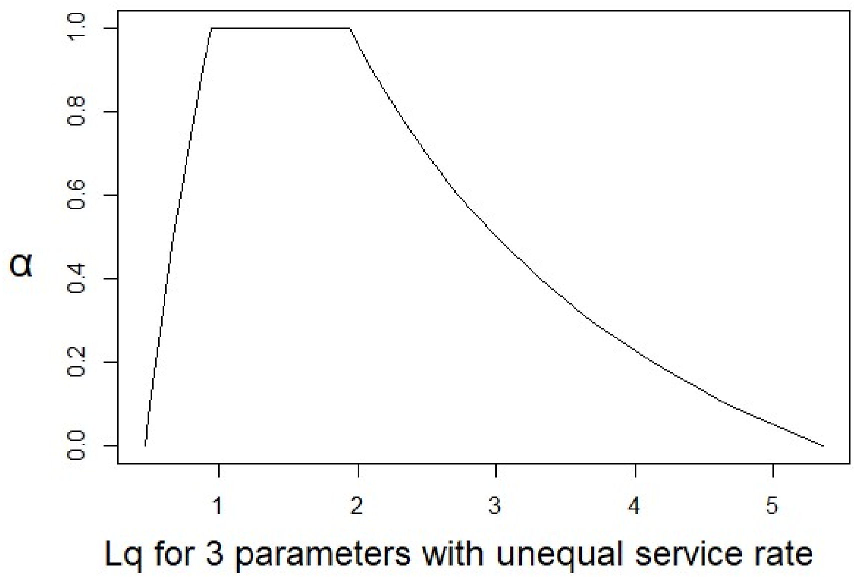

Fig. 10 The membership function of the Trapezoidal Expected queue length Lq of three parameters with corresponding arrival and service rate

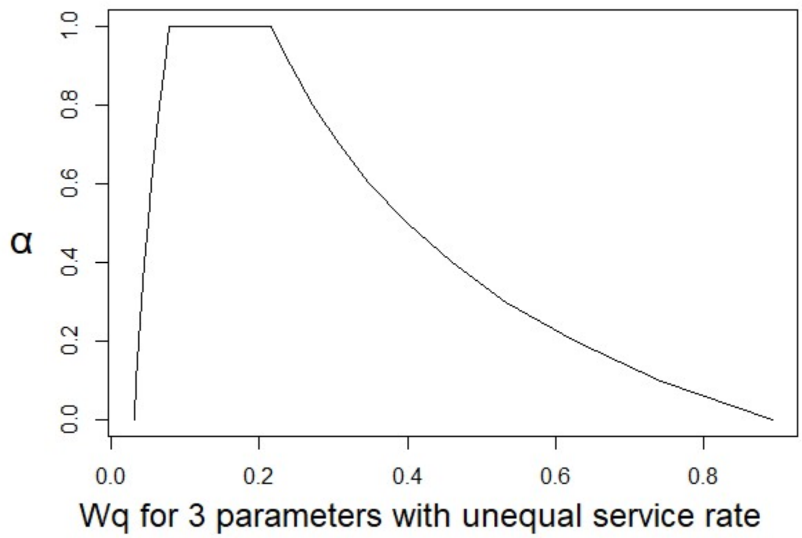

Fig. 11 The membership function of the TrapezoidalExpected waiting time in the queue Wqof three Parameters with corresponding arrival and service rate

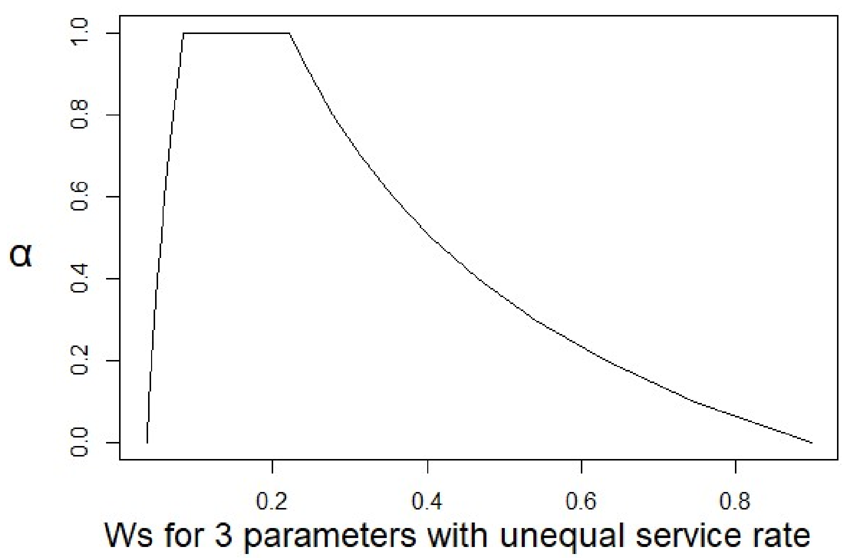

Fig.12 The membership function of the TrapezoidalExpectedwaiting time in the

system Ws of three Parameters with corresponding arrival and service rate

5.Discussions

We implement α-cuts of the batch arrival size, arrival rate, service rate, and fuzzy estimated number at various levels (0, 0.1, 0.2, etc.) using R programming. Table 1 displays several intervals for fuzzy performance measures in batch queues at different probability levels for customers with two parameters.

Table 1 gives a clear explanation of the waiting line length, fuzzy projected wait times in the line, and systemic wait times at various probability levels for the customer batch arrivals. Here is a list of the 22 values of α= 0; 0.1; 0.2; 0.3; 1.0. Figure 1 depicts the approximate shape of Lq for two parameters; Wq and Ws are produced from these 22, values in Figures 2 and 3, respectively. In the end, the rough contour has a nice, continuous function. The α-cut indicates the probability that these performance measures will fall inside the respective range.

Specifically, the performance measures could exist within the range of α= 0 and α= 1 are most likely present within the range. For instance, the most likely value of the expected queue length value of Lq is limited to the interval between 0.3056 and 2.9167. In the exact same way, the waiting time in the queue value Wq values is only allowed to fall between 0.0180 and 0.5833, never below or above that range. The Ws will fall between 0.0407 and 0.6096; it cannot go outside of this range or get higher than 0.6096.

As shown in Table 2, the length of the queue Lq for the three parameters is extremely likely to remain within the range of 0.3070 and 2.4310, without exceeding or falling below it. As a result, the queue Wq's three parameter waiting time is always within this range, never falling below 0.0085 or rising over 0.2723.Furthermore, the wait times for the three parameters in the system Ws are consistently within the range of 0.0160 and 0.2803, never decreasing or increasing.

For all performance measurements of different service rates of corresponding arrival rates for trapezoidal fuzzy numbers in tables 3 and 4, the range between α= 0 and α= 1 is satisfied. Table 3 indicates that for two parameters with corresponding service rates, the estimated queue length value (Lq) is most likely to fall between 0.3439 and 7.1667. It cannot fall outside of this range. In the same manner, Wq's value is restricted to the range of 0.0202 to 1.4333 and cannot slip outside of it. The waiting time in the system might vary from 0.0341 to 1.4500 in the system Ws. On the solution interval [0.3056, 2.9176]⊂[0.3439, 7.1669], compare the two parameter uniform service rate with the two parameter different service rate to the

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

corresponding arrival rate. Consequently, thedifferent service rate to the corresponding arrival rate must have the bigger interval than two parameter uniform service rate.

Table 4 illustrates that the predicted length of the queue (Lq) for three parameters with different service rates can range from 0.4655 to 5.3537 and never goes outside of this range. Comparably, the Wq waiting time for the queue of three parameters and different service rates to the corresponding arrival rate is always between 0.0310 and 0.8923. It never goes beyond this range. As with Ws, the system's waiting time varies between 0.0367 and 0.8982. When developing a queue system with bulk arrivals, the aforementioned information will be quite helpful. The subsequent figures, Figs. 4 to Fig. 12, shows important performance measures.

6.CONCLUSION:

The modelling of a bulk queuing system in triangular fuzzy numbers using α -cut [4] was extended to trapezoidal fuzzy numbers in this study. The model's performance is measured for two and three parameters using trapezoidal fuzzy numbers. Moreover, we evaluated two and three factors using a uniform service rate and different service rate to the corresponding arrival rate model.

Numerical findings were generated using the R -programming software. The findings show the applicability in a range of real-world situations, such as weekend get-togethers with family or friends, college or school canteen break periods, and festival times at textile shops and other establishments. As can be seen from the above tables 2 and table 4, a fuzzy system that exhibits a complete conservation of fuzziness in both the input and output data is also used to express the system's performance measures when the arrival rates $(\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$ and service rates $(\bar{\mu}_1, \bar{\mu}_2, \bar{\mu}_3)$ are expressed in fuzzy numbers.

The method's effectiveness in determining the system performance measure for bulk arrivals with a uniform service rate and a varied service rate from the corresponding arrival rates was demonstrated by the numerical results. Future research into queueing system components that expand multi-channel model examination across constant batch size of arrival rate and constant batch size of service rate should be quite intriguing.

REFERENCES

1. A.Krishnamoorthy and P.V. Ushakumari, "a Queueing system with single arrival bulk service and single departure", Mathematical and computer modelling ,vol.31, (2000),pp99-108.
2. Li,R.J.andLee,E.S.,"Analysisoffuzzyqueues"ComputersandMathematicswithApplication

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

s,17, (1889),pp (1143-1147).
3. Meenu Mittal, T.P. Singh & Deepak Gupta," Threshold effect on a fuzzy queue model withbatcharrival",AryabhattaJournalofMathematics&Informatics,vol-7,No.-1, (2015),pp(109-118).
4. MeenuMittal, T.P.Singh and Deepak Gupta, " Modelling of a bulk queue system in triangular fuzzy numbers using A-cut", IJITE, Vol 4(9), (2016) ,pp.72-79.
5. Negi,D.S.andLee,E.S.,"Analysisandsimulationoffuzzyqueues",Fuzzysets&systems46, (1992),pp(321-330).
6. PourdarvishA.&M.Shokry,"$M^{[X]}$/M/1QueuingSystemwithFuzzyParameters",Pure&AppliedMathematikaSciences,vol.-LXIX,No.1-2, (2009),pp(13-21).
7. R.Shrinivasan,"FuzzyqueueingmodelusingDSWalgorithm",InternationalJournalofAdvancedresearchinMathematicsandApplication,Vol.-1(1),(2014), pp (57-62).
8. T.P. Singh, Kusum & Gupta Deepak, "Fuzzy queue model to interdependent communicationsystemwithbulkarrival",AryabhattaJournalofMathematics&Informatics,vol-4,No.-1,(2012), pp (41-48).
9. ZadahL.A.,"fuzzysetsandsystems",SystemtheoryBrooklynN.Y. (1965),pp29-37.
10. ZadahL.A,"TheconceptofLinguisticvariable&itsapplicationstoApproximateResoning-1"JournalofInformationScience,vol.-8,(1975), pp(199-249).
11. Zadeh's,L.A.,"Fuzzysetsabasisforatheoryofprobability",Fuzzysetsandsystem1, (1978),pp (3-28).

Vol. 21, No. 1, (2024)
ISSN: 1005-0930