FUZZY NETWORK PROJECT PLANNING AND SCHEDULE BY THE CRITICAL PATH TECHNIQUES

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Abstract:-
In this paper, we compute the total project completion time for fuzzy networking conditions where the activity duration is expressed as an octagonal fuzzy integer using the critical path strategy. The application of the octagonal fuzzy number in ambiguous situations is the main topic of this work deals with OCFNs in their modified form. To demonstrate the significant improvement of the proposed methods and methodologies, a relevant numerical example is provided.
2020Mathematics Subject Classification: Primary 03E72, Secondary 90B99
Keywords: Octagonal Fuzzy Number, Critical Path, Fuzzy Project network, Fuzzy ranking

1. Introduction:-

The term "Large-scale, intricate projects can be planned, scheduled, and controlled using network planning and scheduling approaches." is occasionally used to describe network analysis. The logic behind these strategies is that the projects are a network of interdependent operations. A project needs to be completed in a specific amount of time by completing several related tasks, according to a specified sequence, and for the least amount of money and resources. Network representations are widely used for issues related to financial planning, manufacturing, distribution, resource management, and project planning. The Critical Path Method (CPM), which is frequently unrealistic in real-world projects where uncertainty and variability are inherent, is based on precise, deterministic projections for activity durations. Numerous techniques have been developed to include flexibility and probabilistic components in project scheduling in order to address these uncertainties. The fuzzy set notion, first presented by Zadeh [10] in 1965, is essential for dealing with these kinds of circumstances. Dubois and Prade [2] were the first to examine the shortest path problem, which is common in practice and easy to achieve in an effective way. The critical path from many critical paths was found using a trapezoidal fuzzy number-based linear programming technique by Beaula and Vijaya [1]. The generalized trapezoidal fuzzy number was rated by Kamalanathan et al. [5] using the PILOT (Point of Intersection of Legs of Trapezium) ranking algorithm. Generalized Quadrilateral Fuzzy Numbers were used by Stephen Dinagar, D. and Christopher Raj, B. [9] to solve assignment problems. The uncertain duration of a trapezoidal number that is uncertain was converted into a crisp number in the work of Revathi and Saravanan [6]. The Yager ranking approach is used to determine the degree of membership of the fuzzy complete time duration in Shih-Pin Chen's [7] critical path

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ISSN: 1005-0930

scheme. Stephen Dinagar, D. and Abirami, D. [8] described a technique for determining important paths using minimum path length utilizing interval-valued fuzzy networks. Johnson [4] defined Octagonal Fuzzy Number (OFN) with an emphasis on octagonal fuzzy matrices, in the same manner as he defined other fuzzy numbers. The critical path was identified by Ghousia Begum [3] et al. using the interval valued hexagonal fuzzy number.

The critical path approach can solve complex project networks if the decision parameters are well-defined. However, in reality, decision makers face difficulties in determining the exact duration of project operations due to ambiguous information and differing management conditions. As such, the project networking problem cannot be solved in the traditional, classical manner. The 2nd section covers the tentative and fuzzy discussions. In Section 3, the revised form of OCFN for arithmetic operations is derived. A collection of rankings and procedures are shown in part 4. Section 5 offers an illustration based on recommended methodologies, and Section 6 concludes.

2. Preliminaries

Definition 2.1

A fuzzy set $\tilde{F}$ in X is a set of order pair defined by, $\tilde{F}(x) = \{x, \mu_{\tilde{F}}(x), x \in X, \mu_{\tilde{F}}(x) \in [0,1]\}$, where $\mu_{\tilde{F}}(x)$ is a membership function.

Definition 2.2

A fuzzy set $\tilde{F}$ defined on a set of real number R is said to be a fuzzy number, if its membership function $\tilde{F} : \mathrm{R} \to [0,1]$ has the following characteristic

i) $\tilde{F}$ is convex, ie $\tilde{F}\{\lambda x_1 + (1-\lambda)x_2\} \geq min\{\tilde{F}(x_1), \tilde{F}(x_2)\}$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$

ii) $\tilde{F}$ is normal, i.e. there exist an $x \in R \ni \tilde{F}(x) = 1$

iii) $\tilde{F}$ is a piecewise continuous.

Definition 2.3

A fuzzy number $\tilde{F}$ is octagonal fuzzy number denoted by $\tilde{F} = (f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8)$, the membership function is

$$\mu_{\tilde{F}}(x) = \begin{cases} 0 & , x \leq f_1 \\ \lambda\left(\dfrac{x-f_1}{f_2-f_1}\right) & , f_1 \leq x \leq f_2 \\ \lambda & , f_2 \leq x \leq f_3 \\ \lambda + (1-\lambda)\left(\dfrac{x-f_3}{f_4-f_3}\right) & , f_3 \leq x \leq f_4 \\ 1 & , f_4 \leq x \leq f_5 \\ \lambda + (1-\lambda)\left(\dfrac{f_6-x}{f_6-f_5}\right) & , f_5 \leq x \leq f_6 \\ \lambda & , f_6 \leq x \leq f_7 \\ \lambda\left(\dfrac{f_8-x}{f_8-f_7}\right) & , f_7 \leq x \leq f_8 \\ 0 & , x \geq f_8 \end{cases}$$

, where $0 < \lambda < 1$

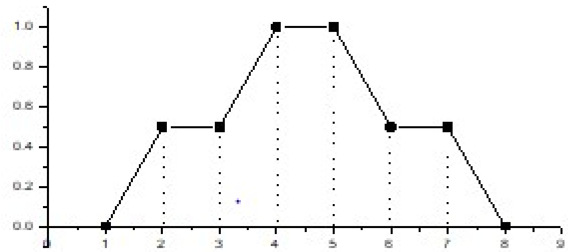

Fig 1.1: Graphical Representation of OCFN

The modified OCFN is represented as $\tilde{F} = \left(M, W, f_{Ls_1}, f_{Ls_2}, f_{Ls_3}, f_{Rs_1}, f_{Rs_2}, f_{Rs_3}\right)$,
where $M = \frac{f_1+f_3+f_5+f_7}{4}$, $W = \frac{(f_5+f_6+f_7)-(f_2+f_3+f_4)}{2}$, $f_{Ls_1} = f_4 - f_1$, $f_{Ls_2} = f_3 - f_1$, $f_{Ls_3} = f_2 - f_1$,
$f_{Rs_1} = f_8 - f_5$, $f_{Rs_2} = f_7 - f_5$, $f_{Rs_3} = f_6 - f_5$

2.1 Ranking function

The ranking function $\mathcal{R}: F(R) \to R$ is defined by $R(\tilde{F})=\left[\frac{f_4+f_5}{2}, \frac{f_{Rs_1}-f_{Ls_1}}{4}\right]$. A OCFN $\tilde{F}$ is +ve, iff $\mathcal{R}(\tilde{F}) > 0$. If $\tilde{F} = (f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8)$, $\tilde{G} = (g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8)$ be any two fuzzy number, then

i. $R(\tilde{F}) \geq R(\tilde{G})$ iff $\tilde{F} \geq \tilde{G}$
ii. $R(\tilde{F}) \leq R(\tilde{G})$ iff $\tilde{F} \leq \tilde{G}$
iii. $R(\tilde{F}) = R(\tilde{G})$ iff $\tilde{F} = \tilde{G}$

2.2 Arithmetic Operations

For any two OCFN $\tilde{F} = \left(M(f), W(f), f_{Ls_1}, f_{Ls_2}, f_{Ls_3}, f_{Rs_1}, f_{Rs_2}, f_{Rs_3}\right)$ and
$\tilde{G} = \left(M(g), W(g), g_{Ls_1}, g_{Ls_2}, g_{Ls_3}, g_{Rs_1}, g_{Rs_2}, g_{Rs_3}\right)$

i) Addition $\tilde{F} + \tilde{G} = \begin{pmatrix} M(f) + M(g), max\left(W(f), W(g)\right), \\ \max\left(f_{Ls_1}, g_{Ls_1}\right), \max\left(f_{Ls_2}, g_{Ls_2}\right), \max\left(f_{Ls_3}, g_{Ls_3}\right), \\ \max\left(f_{Rs_1}, g_{Rs_1}\right), \max\left(f_{Rs_2}, g_{Rs_2}\right), \max\left(f_{Rs_3}, g_{Rs_3}\right) \end{pmatrix}$

ii) Subtraction $\tilde{F} - \tilde{G} = \begin{pmatrix} M(f) - M(g), max\left(W(f), W(g)\right), \\ \max\left(f_{Ls_1}, g_{Ls_1}\right), \max\left(f_{Ls_2}, g_{Ls_2}\right), \max\left(f_{Ls_3}, g_{Ls_3}\right), \\ \max\left(f_{Rs_1}, g_{Rs_1}\right), \max\left(f_{Rs_2}, g_{Rs_2}\right), \max\left(f_{Rs_3}, g_{Rs_3}\right) \end{pmatrix}$

iii) Addition $\tilde{F} \times \tilde{G} = \begin{pmatrix} M(f) \times M(g), max\left(W(f), W(g)\right), \\ \max\left(f_{Ls_1}, g_{Ls_1}\right), \max\left(f_{Ls_2}, g_{Ls_2}\right), \max\left(f_{Ls_3}, g_{Ls_3}\right), \\ \max\left(f_{Rs_1}, g_{Rs_1}\right), \max\left(f_{Rs_2}, g_{Rs_2}\right), \max\left(f_{Rs_3}, g_{Rs_3}\right) \end{pmatrix}$

iv) Division $\tilde{F} \div \tilde{G} = \begin{pmatrix} M(f) \div M(g), max\left(W(f), W(g)\right), \\ \max\left(f_{Ls_1}, g_{Ls_1}\right), \max\left(f_{Ls_2}, g_{Ls_2}\right), \max\left(f_{Ls_3}, g_{Ls_3}\right), \\ \max\left(f_{Rs_1}, g_{Rs_1}\right), \max\left(f_{Rs_2}, g_{Rs_2}\right), \max\left(f_{Rs_3}, g_{Rs_3}\right) \end{pmatrix}$

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ISSN: 1005-0930

3. Planning for networks and scheduling

3.1 Symbolic Representation

Let's make use of these notes for basic computations.

$AF_{MOFN}(ij)$ = Activity b/w tail event (i) and head event (j)
$E_{MOFN}(ij)$ = Earliest occurrence event time i
$L_{MOFN}(ij)$ = Latest occurrence event time j
$ES_{MOFN}(ij)$ = Earliest start time
$EF_{MOFN}(ij)$ = Earliest finish time
$LS_{MOFN}(ij)$ = Latest start time
$LF_{MOFN}(ij)$ = Latest finish time
$TF_{MOFN}(ij)$ = Total float time

3.2 Properties

i) $ES_{MOFN}(ij) = Max\ \{ES_{MOFN}(ij) + AF_{MOFN}(ij)\}$
ii) $LS_{MOFN}(ij) = Min\{LS_{MOFN}(ij) - AF_{MOFN}(ij)\}$
iii) $TF_{MOFN}(ij) = [LF_{MOFN}(ij) - LS_{MOFN}(ij)] - AF_{MOFN}(ij)$

3.3 Algorithm

Step 1: Determine the project network's activity.
Step 2: Use the ranking function to ascertain the relationships between each activity.
Step 3: Design a project network diagram with fuzzy times for activities.
Step 4: Get the earliest, latest fuzzy event time by using property (i),(ii)
Step 5: To calculate the total float of the each activity using property (iii)
Step 5: Identify critical activity, if $TF_{MOFN}(ij) = 0$
Step 6: Link every fuzzy-critical activity between the source and destination nodes.

4. Illustration

Assume a fuzzy networking context where each action's duration is an octagonal fuzzy number.

Table 4.1 - Activity duration of OCFN

Fuzzy	Activity	1-2	1-3	2-4
	duration	(25,26,28,30,32,33,35,36)	(38,40,42,44,46,48,50,52)	(35,38,39,42,43,45,47,49)
	Activity	4-5	3-4	4-6
	duration	(32,34,38,41,44,47,50,53)	(20,24,26,29,32,35,38,40)	(40,42,44,46,48,50,52,54)
	Activity	4-7	5-7	6-7
	duration	(42,44,48,51,53,55,57,59)	(59,61,63,65,67,69,71,72)	(28,30,32,34,36,38,40,42)

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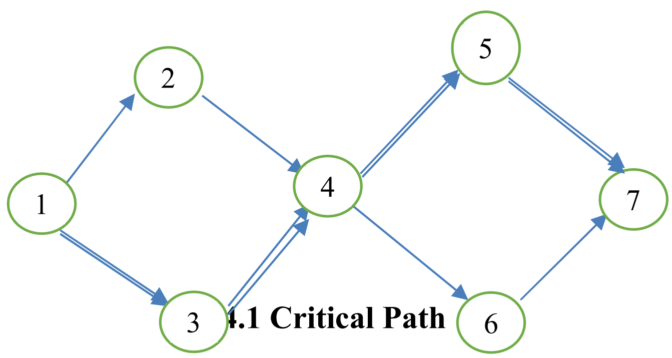
Table 4.2 – Modified form of OCFN

Fuzzy	Activity	1-2	1-3	2-4
	duration	(30,8,5,3,2,4,3,1)	(44,9,6,4,2,6,4,2)	(41,8,7,5,3,6,4,2)
	Activity	4-5	3-4	4-6
	duration	(41,13,9,6,3,9,6,3)	(29,14,9,6,3,8,5,2)	(46,9,6,4,2,6,4,2)
	Activity	4-7	5-7	6-7
	duration	(50,10,9,6,3,6,4,2)	(66,9,6,4,2,5,3,1)	(34,9,6,4,2,6,4,2)

Table 4.3 – Earliest, Latest time of OCFN

Activity	**Fuzzy**			
	Earliest		**Latest**	
	Start	**Finish**	**Start**	**Finish**
1-2	(0,0,0,0,0,0,0,0)	(30,8,5,3,2,4,3,1)	(2,14,9,6,3,8,5,2)	(32,14,9,6,3,8,5,2)
1-3	**(0,0,0,0,0,0,0,0)**	**(44,9,6,4,2,6,4,2)**	**(0,9,6,4,2,6,4,2)**	**(44,9,6,4,2,6,4,2)**
2-4	(30,8,5,3,2,4,3,1)	(71,8,7,5,3,6,4,2)	(32,14,9,6,3,8,5,2)	(73,14,9,6,3,8,5,2)
3-4	**(44,9,6,4,2,6,4,2)**	**(73,14,9,6,3,8,5,2)**	**(44,14,9,6,3,8,5,2)**	**(73,14,9,6,3,8,5,2)**
4-5	**(73,14,9,6,3,8,5,2)**	**(114,14,9,6,3,9,6,3)**	**(73,14,9,6,3,8,5,2)**	**(114,14,9,6,3,9,6,3)**
4-6	(73,14,9,6,3,8,5,2)	(119,14,9,6,3,8,5,2)	(100,14,9,6,3,9,6,3)	(146,14,9,6,3,9,6,3)
4-7	(73,14,9,6,3,8,5,2)	(123,14,9,6,3,8,5,2)	(130,14,9,6,3,9,6,3)	(180,14,9,6,3,9,6,3)
5-7	**(114,14,9,6,3,9,6,3)**	**(180,14,9,6,3,9,6,3)**	**(114,14,9,6,3,9,6,3)**	**(180,14,9,6,3,9,6,3)**
6-7	(119,14,9,6,3,8,5,2)	(153,14,9,6,3,8,5,2)	(146,14,9,6,3,9,6,3)	(180,14,9,6,3,9,6,3)

The critical path is 1-3-4-5-7



.1 Critical Path

Result:-

Fuzzy crucial activities are found, and the critical path in network of projects is formed by joining these important tasks. This critical path's length provides an estimate of how long the project will take to finish overall. The challenge states that the octagonal fuzzy number (180,14,9,6,3,9,6,3), which represents the fuzzy project's total completion time, must be fulfilled.

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5. Conclusion

In this study, we developed a novel critical path approach that uses an octagonal fuzzy number to determine the best time to solve a fuzzy networking problem yet retains the nature of the task into a traditional networking problem. The numerical example demonstrates unequivocally how our suggested methodology outperforms current techniques and offers a better solution to the fuzzy networking issue. By working directly with octagonal fuzzy numbers, our method successfully accounts for the uncertainty and unpredictability included in project timeframes. This improves project management and planning in situations when uncertainty is inevitable.

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