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BIPOLAR VAGUE α GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In this paper a new concept of bipolar vague closed sets called bipolar vague α generalized closed sets are introduced and their properties are thoroughly studied and analyzed. Some new interesting propositions based on the newly introduced set are presented.
Keywords: Bipolar vague sets, bipolar vague topology, bipolar vague α generalized closed sets.

1. Introduction

Fuzzy set was introduced by L.A.Zadeh [10] in 1965. The concept of fuzzy topology was introduced by C.L.Chang [3] in 1968. The generalized closed sets in general topology were first introduced by N.Levine [9] in 1970. K.Atanassov [2] in 1986 introduced the concept of intuitionistic fuzzy sets. The notion of vague set theory was introduced by W.L.Gau and D.J.Buehrer [7] in 1993. D.Coker [6] in 1997 introduced intuitionistic fuzzy topological spaces. Bipolar- valued fuzzy sets, which was introduced by K.M.Lee [8] in 2000 is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1,1]. A new class of generalized bipolar vague sets was introduced by S.Cicily Flora and I.Arockiarani [4] in 2016. The purpose of this paper is to introduce and analyze the concept of bipolar vague closed sets called bipolar vague α generalized closed sets.

2. Preliminaries

Here in this paper the bipolar vague topological spaces are denoted by (X, BV_τ). Also, the bipolar vague interior, bipolar vague closure of a bipolar vague set A are denoted by BVInt(A) and BVCl(A). The complement of a bipolar vague set A is denoted by A^c and the empty set and whole sets are denoted by $0_\sim$ and $1_\sim$ respectively.

Definition 2.1: [8] Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is μ_A^+: X→ [0,1], and a negative membership function μ_A^-, that is μ_A^-: X→ [-1,0]. For the sake of simplicity, we shall use the symbol A = $\{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.2: [8] Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

(i) $\mu_{A\cup B}^+$= max $\{\mu_A^+(x), \mu_B^+ x)\}$

(ii) $\mu^{-}_{A\cup B}$= min $\{\mu^{-}_{A}(x), \mu^{-}_{B}x)\}$
(iii) $\mu^{+}_{A\cap B}$= min $\{\mu^{+}_{A}(x), \mu^{+}_{B}x)\}$
(iv) $\mu^{-}_{A\cap B}$= max $\{\mu^{-}_{A}(x), \mu^{-}_{B}x)\}$
(v) $\mu^{+}_{A^c}$(x) = 1-$\mu^{+}_{A}(x)$ and $\mu^{-}_{A^c}$(x) = -1-$\mu^{-}_{A}(x)$ for all $x \in X$.

Definition 2.3: [7] A vague set A in the universe of discourse U is a pair of $(t_A,\ f_A)$ where t_A: U→[0,1], f_A: U→[0,1] are the mapping such that $t_A + f_A \leq 1$ for all $u \in U$. The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A,\ 1 - f_A]$ is called the vague value of u in A, and denoted by $\nu_A(u)$, that is $\nu_A(u) = [t_A(u), 1 - f(u)]$.

Definition 2.4: [7] Let A be a non-empty set and the vague set A and B in the form A = $\{\langle x, t_A(x), 1 - f_A(x)\rangle$: x $\in$ X }, B = $\{\langle x, t_B(x), 1 - f_B(x)\rangle$: x $\in$ X }. Then
(i) A ⊆ B if and only if $t_A(x) \leq t_B(x)$ and $1 - f_A(x) \leq 1 - f_B(x)$
(ii) A ∪ B = $\left\{\langle \max\left(t_A(x), t_B(x)\right), \max\left(1 - f_A(x), 1 - f_B(x)\right)\rangle / x \in X \right\}$.
(iii) A ∩ B = $\left\{\langle \min\left(t_A(x), t_B(x)\right), \min\left(1 - f_A(x), 1 - f_B(x)\right)\rangle / x \in X \right\}$.
(iv) A^c = $\{\langle x, f_A(x), 1 - t_A(x)\rangle$: x $\in$ X }.

Definition 2.5: [1] Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form A = $\{\langle x, [t^{+}_{A}(x), 1 - f^{+}_{A}(x)], [-1 - f^{-}_{A}(x), t^{-}_{A}(x)]\rangle$: x $\in$ X } where $[t^{+}_{A}, 1 - f^{+}_{A}]$: X→[0,1] and $[-1 - f^{-}_{A}, t^{-}_{A}]$: X→[-1,0] are the mapping such that $t^{+}_{A}(x) + f^{+}_{A}(x) \leq 1$ and -1≤ t^{-}_{A}+ f^{-}_{A}. The positive membership degree $[t^{+}_{A}(x), 1 - f^{+}_{A}(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f^{-}_{A}(x), t^{-}_{A}(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set ν^{+}_{A} = $[t^{+}_{A}, 1 - f^{+}_{A}]$ and ν^{-}_{A} = $[-1 - f^{-}_{A}, t^{-}_{A}]$.

Definition 2.6: [5] A bipolar vague set A = $[\nu^{+}_{A}, \nu^{-}_{A}]$ of a set U with ν^{+}_{A}= 0 implies that t^{+}_{A} = 0, $1 - f^{+}_{A}$= 0 and ν^{-}_{A}= 0 implies that t^{-}_{A} = 0, $-1 - f^{-}_{A}$ = 0 for all x ∈ U is called zero bipolar vague set and it is denoted by 0.

Definition 2.7: [5] A bipolar vague set A = $[\nu^{+}_{A}, \nu^{-}_{A}]$ of a set U with ν^{+}_{A}= 1 implies that t^{+}_{A} = 1, $1 - f^{+}_{A}$= 1 and ν^{-}_{A}= -1 implies that t^{-}_{A} = -1, $-1 - f^{-}_{A}$ = -1 for all x ∈ U is called unit bipolar vague set and it is denoted by 1.

Definition 2.8: [4] Let A = ⟨x, $[t^{+}_{A}, 1 - f^{+}_{A}], [-1 - f^{-}_{A}, t^{-}_{A}]$⟩ and ⟨x, $[t^{+}_{B}, 1 - f^{+}_{B}], [-1 - f^{-}_{B}, t^{-}_{B}]$⟩ be two bipolar vague sets then their union, intersection and complement are defined as follows:
(i) A ∪ B = {⟨x, $[t^{+}_{A\cup B}(x), 1 - f^{+}_{A\cup B}(x)], [-1 - f^{-}_{A\cup B}(x), t^{-}_{A\cup B}(x)]\rangle / x \in X$ } where
$t^{+}_{A\cup B}(x)$ = max $\{t^{+}_{A}(x), t^{+}_{B}(x)\}$, $t^{-}_{A\cup B}(x)$ = min $\{t^{-}_{A}(x), t^{-}_{B}(x)\}$ and
$1 - f^{+}_{A\cup B}(x)$ = max $\{1 - f^{+}_{A}(x), 1 - f^{+}_{B}(x)\}$,
$-1 - f^{-}_{A\cup B}(x)$ = min $\{-1 - f^{-}_{A}(x), -1 - f^{-}_{B}(x)\}$.
(ii) A ∩ B = {⟨x, $[t^{+}_{A\cap B}(x), 1 - f^{+}_{A\cap B}(x)], [-1 - f^{-}_{A\cap B}(x), t^{-}_{A\cap B}(x)]\rangle / x \in X$ } where
$t^{+}_{A\cap B}(x)$ = min $\{t^{+}_{A}(x), t^{+}_{B}(x)\}$, $t^{-}_{A\cap B}(x)$ = max $\{t^{-}_{A}(x), t^{-}_{B}(x)\}$ and
$1 - f^{+}_{A\cap B}(x)$ = min $\{1 - f^{+}_{A}(x), 1 - f^{+}_{B}(x)\}$,

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$-1 - f^{-}_{A\cup B}(x)$ = max $\{-1 - f^{-}_{A}(x), -1 - f^{-}_{B}(x)\}$.

(iii) $A^c = \{\langle x, [f^{+}_{A}(x), 1 - t^{+}_{A}(x)], [-1 - t^{-}_{A}(x), f^{-}_{A}(x)]\rangle / x \in X\}$.

Definition 2.9: [4] Let A and B be two bipolar vague sets defined over a universe of discourse X. We say that A ⊆ B if and only if $t^{+}_{A}(x) \leq t^{+}_{B}(x)$, $1 - f^{+}_{A}(x) \leq 1 - f^{+}_{B}(x)$ and $t^{-}_{A}(x) \geq t^{-}_{B}(x)$, $-1 - f^{-}_{A}(x) \geq 1 - f^{-}_{B}(x)$ for all x ∈ X.

Definition 2.10: [4] A bipolar vague topology (BVT) on a non-empty set X is a family BV_τ of bipolar vague set in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in BV_\tau$

(ii) $G_1 \cap G_2 \in BV_\tau$, for any $G_1, G_2 \in BV_\tau$

(iii) $\cup G_i \in BV_\tau$, for any arbitrary family $\{G_i : G_i \in BV_\tau, i \in I\}$.

In this case the pair (X, BV_τ) is called a bipolar vague topological space and any bipolar vague set (BVS) in BV_τ is known as bipolar vague open set in X. The complement A^c of a bipolar vague open set (BVOS) A in a bipolar vague topological space (X, BV_τ) is called a bipolar vague closed set (BVCS) in X.

Definition 2.11: [4] Let (X, BV_τ) be a bipolar vague topological space $A = \langle x, [t^{+}_{A}, 1 - f^{+}_{A}], [-1 - f^{-}_{A}, t^{-}_{A}]\rangle$ be a bipolar vague set in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

BVInt(A) = ∪ {G: G is a bipolar vague open set in X and G ⊆ A},

BVCl(A) = ∩ {K: K is a bipolar vague closed set in X and A⊆ K}.

Note that BVCl(A) is a bipolar vague closed set and BVInt(A) is a bipolar vague open set in X. Further,

(i) A is a bipolar vague closed set in X if and only if BVCl(A) = A,

(ii) A is a bipolar vague open set in X if and only if BVInt(A) = A.

Definition 2.12: [4] Let (X, BV_τ) be a bipolar vague topological space. A bipolar vague set A in (X, BV_τ) is said to be a generalized bipolar vague closed set if BVCl(A) ⊆ G whenever A⊆ G and G is bipolar vague open. The complement of a generalized bipolar vague closed set is generalized bipolar vague open set.

Definition 2.13: [4] Let (X, BV_τ) be a bipolar vague topological space and A be a bipolar vague set in X. Then the generalized bipolar vague closure and generalized bipolar vague interior of A are defined by,

GBVCl(A) = ∩ {G: G is a generalized bipolar vague closed set in X and A⊆ G},

GBInt(A) = ∪ {G: G is a generalized bipolar vague open set in X and A ⊇ G}.

3. Bipolar Vague α Generalized Closed Sets in Topological Spaces

In this section we introduce bipolar vague α closure, bipolar vague α interior and α generalized closed set and its respective open set in bipolar vague topological spaces and discuss some of their properties.

Definition 3.1: A bipolar vague set A of a bipolar vague topological space X, is said to be

(i) a bipolar vague α-open set if A ⊆ BVInt(BVCl(BVInt(A)))

(ii) a bipolar vague pre-open set if A ⊆ BVInt(BVCl(A))

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(iii) a bipolar vague semi-open set if A ⊆ BVCl(BVInt(A))
(iv) a bipolar vague semi-α-open set if A ⊆ BVCl(αBVInt(A))
(v) a bipolar vague regular-open set BVInt(BVCl(A)) = A
(vi) a bipolar vague β-open set A ⊆ BVCl(BVInt(BVCl(A))).

Definition 3.2: A bipolar vague set A of a bipolar vague topological space X, is said to be
(i) a bipolar vague α-closed set if BVCl(BVInt(BVCl(A))) ⊆ A
(ii) a bipolar vague pre-closed set if BVCl(BVInt(A)) ⊆ A
(iii) a bipolar vague semi-closed set if BVInt(BVCl(A)) ⊆ A
(iv) a bipolar vague semi-α-closed set if BVInt(αBVCl(A)) ⊆ A
(v) a bipolar vague regular-closed set if BVCl(BVInt(A)) = A
(vi) a bipolar vague β-closed set if BVInt(BVCl(BVInt(A))) ⊆ A.

Definition 3.3: Let A be a bipolar vague set of a bipolar vague topological space (X, BV_τ). Then the bipolar vague α interior and bipolar vague α closure are defined as

BV_αInt(A) = ∪ {G: G is a bipolar vague α-open set in X and G ⊆ A},

BV_αCl(A) = ∩ {K: K is a bipolar vague α-closed set in X and A⊆ K}.

Result 3.4: Let A be a bipolar vague set in X. Then BV_αCl(A) = A∪BVCl(BVInt(BVCl(A))).

Proof: Since BV_αCl(A) is a bipolar vague α-closed set. BVCl(BVInt(BVCl(BV_αCl(A)))) ⊆ BV_αCl(A) and A∪BVCl(BVInt(BVCl(A))) ⊆ A∪ BVCl(BVInt(BVCl(BV_αCl(A)))) ⊆ A∪ BV_αCl(A) = BV_αCl(A)--------------(i).

Now, BVCl(BVInt(BVCl(A∪BVCl(BVInt(BVCl(A))))) ⊆ BVCl(BVInt(BVCl(A∪BVCl(A)))) = BVCl(BVInt(BVCl(BVCl(A)))) = BVCl(BVInt(BVCl(A))) ⊆ A∪BVCl(BVInt(BVCl(A))). Therefore A∪BVCl(BVInt(BVCl(A))) is a bipolar vague α-closed set in X and hence BV_αCl(A) ⊆ A∪BVCl(BVInt(BVCl(A)))-----------(ii).

From (i) and (ii), BV_αCl(A) = A∪BVCl(BVInt(BVCl(A))).

Definition 3.5: A bipolar vague set A in a bipolar vague topological space X, is said to be a bipolar vague α generalized closed set if BV_αCl(A) ⊆ U whenever A⊆ U and U is a bipolar vague open set in X. The complement A^c of a bipolar vague α generalized closed set A is a bipolar vague α generalized open set in X.

Example 3.6: Let X = {a,b} and τ = {$0_\sim$, A, B, $1_\sim$} where A = ⟨x, [0.5, 0.6] [-0.6, -0.6], [0.6, 0.9] [-0.6, -0.5]⟩ and B = ⟨x, [0.4, 0.5] [-0.5, -0.5], [0.4, 0.6] [-0.5, -0.4]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.6] [-0.5, -0.4], [0.4, 0.5] [-0.4, -0.3]⟩ be any bipolar vague set in X. Then M ⊆ A where A is a bipolar vague open set in X. Now BV_αCl(M) = M ∪ B^c = B^c ⊆ A. Therefore, M is a bipolar vague α generalized closed set in X.

Proposition 3.7: Every bipolar vague closed set A is a bipolar vague α generalized closed set in X but not conversely in general.

Proof: Let A ⊆ U where U is a bipolar vague open set in X. Now, BV_αCl(A) = A ∪ BVCl(BVInt(BVCl(A))) ⊆ A ∪ BVCl(A) = A ∪ A = A ⊆ U, by hypothesis. Hence A is a bipolar vague α generalized closed set in X.

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Example 3.8: In Example 3.6, M is a bipolar vague α generalized closed set in X but not a bipolar vague closed set in X as BVCl(M) = $B^c \neq$ M.

Remark 3.9: Every bipolar vague semi-closed set and every bipolar vague α generalized closed set in a bipolar vague topological space X are independent to each other in general.

Example 3.10: In Example 3.6, M is a bipolar vague α generalized closed set in X but not a bipolar vague semi-closed set as BVInt(BVCl(M)) = B $\not\subset$ M.

Example 3.11: Let X = {a,b} and τ = {$0_\sim$, A, B, C, $1_\sim$} where A = ⟨x, [0.5, 0.5] [-0.5, -0.5], [0.4, 0.4] [-0.3, -0.2]⟩, B = ⟨x, [0.8, 0.8] [-0.8, -0.8], [0.7, 0.7] [-0.8, -0.8]⟩ and C = ⟨x, [0.2, 0.2] [-0.2, -0.2], [0.1, 0.1] [-0.2, -0.2]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.5] [-0.2, -0.3], [0.3, 0.3] [-0.2, -0.2]⟩ be any bipolar vague set in X. Then M is a bipolar vague semi-closed set but not a bipolar vague α generalized closed set as M ⊆ A, B and BV_αCl(M) = M ∪ A^c = $A^c \not\subset$ A.

Remark 3.12: Every bipolar vague pre-closed set and every bipolar vague α generalized closed set in a bipolar vague topological space X are independent to each other in general.

Example 3.13: In Example 3.11, M is a bipolar vague pre-closed set but not a bipolar vague α generalized closed set as seen in the respective example.

Example 3.14: Let X = {a,b} and τ = {$0_\sim$, A, B, $1_\sim$} where A = ⟨x, [0.8, 0.5] [-0.5, -0.5], [0.7, 0.9] [-0.5, -0.5]⟩ and B = ⟨x, [0.5, 0.2] [-0.5, -0.5], [0.1, 0.3] [-0.5, -0.5]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.5, 0.3] [-0.5, -0.5], [0.2, 0.3] [-0.5, -0.5]⟩ be any bipolar vague set in X. Then M is a bipolar vague α generalized closed set in X but not a bipolar vague pre-closed set as BVCl(BVInt(M)) = $B^c \not\subset$ M.

Proposition 3.15: Every bipolar vague α-closed set A is a bipolar vague α generalized closed set in X but not conversely in general.

Proof: Let A ⊆ U, where U is a bipolar vague open set in X. Now BV_αCl(A) = A ∪ BVCl(BVInt(BVCl(A))) ⊆ A ∪ A = A ⊆ U, by hypothesis. Hence A is a bipolar vague α generalized closed set in X.

Example 3.16: In Example 3.6, M is a bipolar vague α generalized closed set in X but not bipolar vague α-closed set as BVCl(BVInt(BVCl(M))) = $B^c \not\subset$ M.

Proposition 3.17: Every bipolar vague open set, bipolar vague α-open set are bipolar vague α generalized open set but not conversely in general.

Proof: Obvious.

Example 3.18: In Example 3.6, M^c is a bipolar vague α generalized open set in X but not bipolar vague open set, bipolar vague α-open set in X.

Remark 3.19: Both bipolar vague semi-open set and bipolar vague pre-closed set are independent to bipolar vague α generalized open set in X in general.

Example 3.20: The above remark can be proved easily from the examples 3.10, 3.11 and 3.13, 3.14 respectively.

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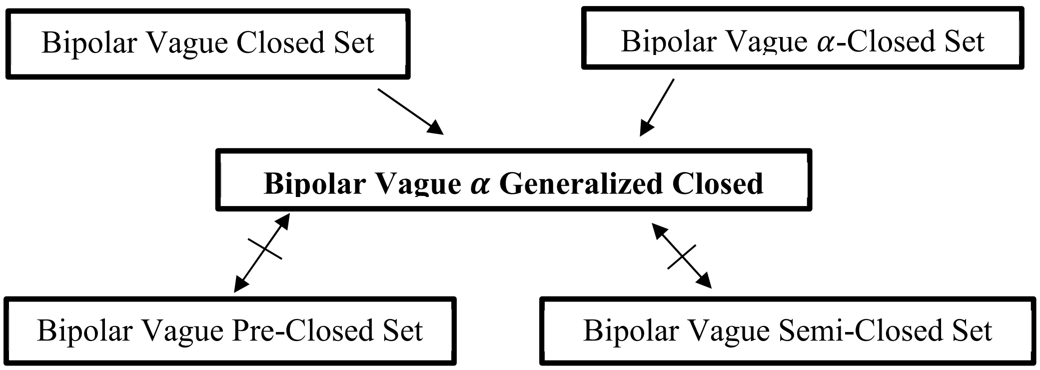
The following diagram implications are true:



Proposition 3.21: The union of any two bipolar vague α generalized closed sets is a bipolar vague α generalized closed set in a bipolar vague topological space X.

Proof: Let A and B be any two bipolar vague α generalized closed sets in a bipolar vague topological space X. Let A ∪ B ⊆ U where U is a bipolar vague open set in X. Then A ⊆ U and B ⊆ U. Now BV_αCl(A ∪ B) = (A ∪ B) ∪ BVCl(BVInt(BVCl(A ∪ B))) ⊆ (A ∪ B) ∪ BVCl((BVCl(A ∪ B)) ⊆ (A ∪ B) ∪ BVCl(A ∪ B) ⊆ BVCl(A ∪ B) ⊆ BVCl(A) ∪ BVCl(B) ⊆ U ∪ U = U, by hypothesis. Hence A ∪ B is a bipolar vague α generalized closed set in X.

Remark 3.22: The intersection of any two bipolar vague α generalized closed sets need not be a bipolar vague α generalized closed set in a bipolar vague topological space X.

Example 3.23: Let X = {a,b} and τ = {$0_\sim$, A, B, $1_\sim$} where A = ⟨x, [0.5, 0.5] [-0.6, -0.4], [0.4, 0.4] [-0.3, -0.2]⟩ and B = ⟨x, [0.8, 0.8] [-0.3, -0.2], [0.7, 0.7] [-0.3, -0.2]⟩. Then τ is a bipolar vague topology. Let M = ⟨x, [0.6, 0.6] [-0.3, -0.2], [0.9, 0.9] [-0.3, -0.2]⟩ and N = ⟨x, [0.9, 0.9] [-0.3, -0.2], [0.7, 0.7] [-0.3, -0.2]⟩. Then M and N are bipolar vague α generalized closed sets in X but M ∩ N = ⟨x, [0.6, 0.6] [-0.3, -0.2], [0.7, 0.7] [-0.3, -0.2]⟩ is not a bipolar vague α generalized closed set as M ∩ N ⊆ B and BV_αCl(M ∩ N) =$1_\sim$ ⊄ A.

Proposition 3.24: Let (X, BV_τ) be a bipolar vague topological space. Then for every A belongs to bipolar vague α generalized closed set in X and for every B belongs to bipolar vague set in X. A ⊆ B⊆ BV_αCl(A) implies B belongs to bipolar vague α generalized closed set in X.

Proof: Let B ⊆ U and U be a bipolar vague open set in (X, BV_τ). Then since A ⊆ B. A ⊆ U. By hypothesis, B⊆ BV_αCl(A). Therefore, BV_αCl(B) ⊆ BV_αCl(BV_αCl(A)) = BV_αCl(A) ⊆ U, since A is a bipolar vague α generalized closed set in (X, BV_τ). Hence B belongs to bipolar vague α generalized closed set in X.

Proposition 3.25: If A is a bipolar vague open set and a bipolar vague α generalized closed set in (X, BV_τ), then A is a bipolar vague α-closed set in (X, BV_τ).

Proof: Since A ⊆ A and A is a bipolar vague open set in (X, BV_τ), by hypothesis, BV_αCl(A) ⊆ A. But A ⊆ BV_αCl(A). Therefore, BV_αCl(A) = A. Hence A is a bipolar vague α-closed set in (X, BV_τ).

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Proposition 3.26: Let $(\mathrm{X}, \mathrm{B}V_\tau)$ be a bipolar vague topological space. Then every bipolar vague set in $(\mathrm{X}, \mathrm{B}V_\tau)$ is a bipolar vague α generalized closed set in $(\mathrm{X}, \mathrm{B}V_\tau)$ if and only if bipolar vague α-open set in X equals bipolar vague α-closed set in X.

Proof: Necessity: Suppose that every bipolar vague set in $(\mathrm{X}, \mathrm{B}V_\tau)$ is a bipolar vague α generalized closed set in (X, $\mathrm{B}V_\tau$). Let U belongs to bipolar vague open set in X. Then U belongs to bipolar vague α-open set in X and by hypothesis, $\mathrm{B}V_\alpha$Cl(U) ⊆ U ⊆ $\mathrm{B}V_\alpha$Cl(U). This implies $\mathrm{B}V_\alpha$Cl(U) = U. Therefore, U belongs to bipolar vague α-closed set in X. Hence bipolar vague α-open set in X contained in bipolar vague α-closed set in X. Let A belongs to bipolar vague α-closed set in X. Then A^c belongs to bipolar vague α-open set in X contained in bipolar vague α-closed set in X. That is A^c belongs to bipolar vague α-closed set in X. Therefore, A belongs to bipolar vague α-open set in X. Hence bipolar vague α-closed set in X contained in bipolar vague α-open set in X. Thus, bipolar vague α-open set in X equals bipolar vague α-closed set in X.

Sufficiency: Suppose that bipolar vague α-open set in X equals bipolar vague α-closed set in X. Let A ⊆ U and U be a bipolar vague open set in $(\mathrm{X}, \mathrm{B}V_\tau)$. Then U belongs to bipolar vague α-open set in X and $\mathrm{B}V_\alpha$Cl(A) ⊆ $\mathrm{B}V_\alpha$Cl(U) = U, since U belongs to bipolar vague α-closed set in X, by hypothesis. Therefore, A is a bipolar vague α generalized closed set in X.

Proposition 3.27: If A is a bipolar vague open set and a bipolar vague α generalized closed set in (X, $\mathrm{B}V_\tau$), then A is a bipolar vague regular-open set in (X, $\mathrm{B}V_\tau$).

Proof: Let A be a bipolar vague open set and a bipolar vague α generalized closed set in (X, $\mathrm{B}V_\tau$). Then A is a bipolar vague α-closed set in X. Now BVInt(BVCl(A)) ⊆ BVCl(BVInt(BVCl(A))) ⊆ A. Since A is a bipolar vague open set, A = BVInt(A) ⊆ BVInt(BVCl(A)). Hence BVInt(BVCl(A)) = A and A is a bipolar vague regular-open set in X.

Definition 3.28: A bipolar vague set A in (X, $\mathrm{B}V_\tau$) is a bipolar vague Q-set in X if BVInt(BVCl(A)) = BVCl(BVInt(A)).

Proposition 3.29: For a bipolar vague open set, A in (X, $\mathrm{B}V_\tau$) the following conditions are equivalent:

(i) A is a bipolar vague closed set in (X, $\mathrm{B}V_\tau$),

(ii) A is a bipolar vague α generalized closed set and bipolar vague Q-set in (X, $\mathrm{B}V_\tau$).

Proof: (i) ⇒ (ii). Since A is a bipolar vague closed set, it is a bipolar vague α generalized closed set in (X, $\mathrm{B}V_\tau$). Now, BVInt(BVCl(A)) = BVInt(A) = A = BVCl(A) = BVCl(BVInt(A)), by hypothesis. Hence A is a bipolar vague Q-set in (X, $\mathrm{B}V_\tau$).

(ii) ⇒ (i). Since A is a bipolar vague open set and a bipolar vague α generalized closed set in (X, $\mathrm{B}V_\tau$), by Proposition 3.27, A is a bipolar vague regular-open set in (X, $\mathrm{B}V_\tau$). Therefore, A = BVInt(BVCl(A)) = BVCl(BVInt(A)) = BVCl(A), by hypothesis. Hence A is a bipolar vague closed set in (X, $\mathrm{B}V_\tau$).

Proposition 3.30: Let (X, $\mathrm{B}V_\tau$) be a bipolar vague topological space. Then for every A belongs to bipolar vague α generalized open set in X and for every B belongs to bipolar vague set in X, $\mathrm{B}V_\alpha$Int(A) ⊆ B ⊆ A implies B belongs to bipolar vague α generalized open set in X.

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Proof: Let A be any bipolar vague α generalized open set in X and B be any bipolar vague set in X. By hypothesis, BV_αInt(A) ⊆ B ⊆ A. Then A^c is a bipolar vague α generalized closed set in X and $A^c \subseteq B^c \subseteq$ BV_αCl(A^c). By Proposition 3.24, B^c is a bipolar vague α generalized closed set in (X, BV_τ). Therefore, B is a bipolar vague α generalized open set in (X, BV_τ). Hence B belongs to bipolar vague α generalized open set in X.

Proposition 3.31: Let (X, BV_τ) be a bipolar vague topological space. Then for every A belongs to bipolar vague set in X and for every B belongs to bipolar vague semi-open set in X, B ⊆ A⊆ BVInt(BVCl(B)) implies A belongs to bipolar vague α generalized open set in X.

Proof: Let B be a bipolar vague semi-open set in (X, BV_τ). Then B ⊆ BVCl(BVInt(B)). By hypothesis, A ⊆ BVInt(BVCl(B)) ⊆ BVInt(BVCl(BVCl(BVInt(B)))) ⊆ BVInt(BVCl(BVInt(B))) ⊆ BVInt(BVCl(BVInt(A))). Therefore, A is a bipolar vague α-open set and by Proposition 3.17, A is a bipolar vague α generalized open set in (X, BV_τ). Hence A belongs to bipolar vague α generalized open set in X.

Proposition 3.32: A bipolar vague set A of a bipolar vague topological space (X, BV_τ) is a bipolar vague α generalized open set in (X, BV_τ) if and only if F ⊆ BV_αInt(A) whenever F is a bipolar vague closed set in (X, BV_τ) and F ⊆ A.

Proof: Necessity: Suppose A is a bipolar vague α generalized open set in (X, BV_τ). Let F be a bipolar vague closed set in (X, BV_τ) such that F ⊆ A. Then F^c is a bipolar vague open set and $A^c \subseteq F^c$. By hypothesis A^c is a bipolar vague α generalized closed set in (X, BV_τ), we have BV_αCl(Cl(A)) ⊆ F^c. Therefore, F ⊆ BV_αInt(A).

Sufficiency: Let U be a bipolar vague open set in (X, BV_τ) such that A^c ⊆ U. By hypothesis, U^c ⊆ BV_αInt(A). Therefore, BV_αCl(Cl(A)) ⊆ U and A^c is an bipolar vague α generalized closed set in (X, BV_τ). Hence A is a bipolar vague α generalized open set in (X, BV_τ).

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