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FUZZY OPTIMAL SOLUTION FOR THE SHORTEST MAPPING IN THE UNIVARIATE SEARCH APPROACH USING DECAGONAL FUZZY NUMBERS.

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Abstract

A wide range of fields, including engineering, transportation, almost every type of business, and daily tasks, have made use of fuzzy set theory. Decagonal fuzzy numbers are used in this paper to address issues in univariate search strategies. On the basis of the three new ranking functions, a fresh approach is suggested. This study focuses on a few issues and compares the three distinct ranking functions.

Keywords: Fuzzy number, Decagonal Fuzzy number, Ranking function, Univariate Search method, Fuzzy optimal solution.

1 Introduction

Fuzzy logic was established in 1965 by Lotfi A. Zadeh, a professor of computer science at the University of California, Berkeley. A multivalued logic known as fuzzy logic (FL) establishes intermediate values between binary classifications like true or false, yes or no, high or low, and so on. Computers can express and comprehend concepts like being relatively tall or extremely fast analytically, which enables computer programming to adopt a more human-like thought process. Artificial intelligence, data analysis, decision-making, and other operations research domains are the main uses for ranking fuzzy numbers. Evaluating fuzzy numbers is a crucial part of making decisions in a fuzzy environment.

Bellman and Zadeh [5] first introduced the concept of a fuzzy set as a way to deal with uncertainty that results from imprecision as opposed to randomness. A New Decagonal Fuzzy Number in an Uncertain Linguistic Environment was proposed by Felix and Victor III. Using a decagonal fuzzy number, Nagadevi and Rosario [5] conducted a study on the fuzzy transportation problem. Sikander [7] proposed employing median ranking to get the optimal solution for the fuzzy transportation problem. A Decagonal Fuzzy number was developed by Velmurugan and Subalakshmi [8] as a solution to the Assignment problem using the Hungarian technique. A sign distance algorithm was proposed by Abbasbandy and Asady [1]] in 2006 for sorting fuzzy numbers. In their work, Rajarajeswari et al. [6] introduced a novel operation on fuzzy hexagonal numbers. We introduced ranking fuzzy numbers with interval values by Liou and Wang [4].The topic of fuzzy number comparison was covered by Lee and Li [8]. Fuzzy logic is widely used today in many different applications, such as data sorting and handling, information systems, traffic control systems, washing machines, automatic focus control, automobile engines and automatic gearboxes, air conditioners, data sorting and handling, television video enhancement, and so forth.

This paper, section 2 presents some preliminary information and proposed method using one numerical example and the results are reviewed. Section 3 concludes the paper.

2 Preliminaries and Proposed Method

Definition 2.1. The characteristic function $\mu_{\tilde{D}}$ of a crisp set D ⊂ X assigns a value either 0 or 1 to each individual in the universal set X. This function can be generalized to a function $\mu_{\tilde{D}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{D}} : X \to [0,1]$. The assigned value indicates the membership function and the set $\widetilde{D} = \{(x, \mu_{\tilde{D}}(x)); x \epsilon X\}$ defined by $\mu_{\tilde{D}}(x)$ for $x \epsilon X$ is called fuzzy set.

Definition 2.2. An effective approach for ordering the elements of F(R) is also to define a ranking function $\Re : F(R) \to R$ which maps each fuzzy number into the real line, where a natural order exists. We define orders on F(R) by:

$\tilde{a} \geq \tilde{b}$ if and only if $R(\tilde{a}) \geq R(\tilde{b})$

$\tilde{a} > \tilde{b}$ if and only if $R(\tilde{a}) \geq R(\tilde{b})$

$\tilde{a} = \tilde{b}$ if and only if $R(\tilde{a}) = R(\tilde{b})$

Next, introduced new membership function and ranking function based on the intervals.

Definition 2.3. A fuzzy number $\mu_{\tilde{D}}$ is a decagonal fuzzy number denoted by $\mu_{\widetilde{D1}}$ $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ are real numbers and its membership function $\mu_{\widetilde{D1}}(x)$ is given below based on the intervals [0, 1].

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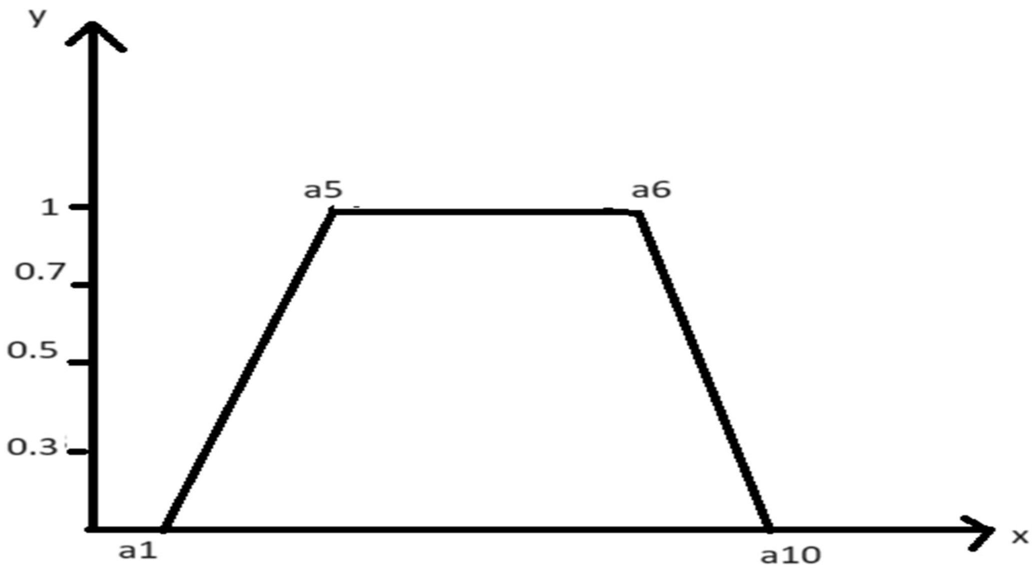

Fig 1 Graphical representation of Decagonal Fuzzy number

$$\mu_{\widetilde{D1}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{3}\left(\frac{x-1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ \frac{1}{3} - \frac{2}{15}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \leq x \leq a_3 \\ \frac{1}{5} - \frac{2}{45}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \\ \frac{1}{7} + \frac{6}{7}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4 \leq x \leq a_5 \\ 1 & a_5 \leq x \leq a_6 \\ 1 - \frac{6}{7}\left(\frac{x-a_6}{a_7-a_6}\right) & a_6 \leq x \leq a_7 \\ \frac{1}{7} + \frac{2}{35}\left(\frac{x-a_7}{a_8-a_7}\right) & a_7 \leq x \leq a_8 \\ \frac{1}{5} + \frac{2}{15}\left(\frac{x-a_8}{a_9-a_8}\right) & a_8 \leq x \leq a_9 \\ \frac{1}{3} + \frac{1}{3}\left(\frac{x-a_9}{a_{10}-a_9}\right) & a_9 \leq x \leq a_{10} \\ 0 & \text{otherwise} \end{cases}$$

and ranking *as* $\Re(\widetilde{D1}) = \frac{a_1+0.3a_2+0.5a_3+0.7a_4+a_5+a_6+0.7a_7+0.5a_8+0.3a_9+a_{10}}{7}$.

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and decagonal fuzzy number defined as $\mu_{\widetilde{D1}} = (L_1\,(u),\ M_1\,(v), N_1\,(w), O_2(t))$ for u ∈ [0, 0.3], v [0.3, 0.5], w [0.5, 0.7], t [0.7, 1] where,

(i) $L_1\,(u)$ is a bounded left continuous non decreasing function over [0, 0.3]
(ii) $M_1\,(u)$ is a bounded left continuous non decreasing function over [0.3, 0.5]
(iii) $N_1\,(w)$ is a bounded left continuous non decreasing function over [0.5, 0.7]
(iv) $O_1\,(t)$ is a bounded left continuous non decreasing function over [0.7, 1]
(v) $L_2\,(u)$ is a bounded continuous non increasing function over [0, 0.3]
(vi) $M_2\,(v)$ is a bounded continuous non increasing function over [0.5, 0.7]
(vii) $N_2(w)$ is a bounded continuous non increasing function over [0.7, 1]
$(viii)$ $O_2\,(t)$ is a bounded continuous non increasing function over [0.7, 1]

Definition 2.4. A fuzzy number $\mu_{\widetilde{D2}}$ is a decagonal fuzzy number denoted by $\mu_{\widetilde{D2}}$ $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ are real numbers and its membership function $\mu_{\widetilde{D2}}(x)$ is given below based on the intervals [0, 1].

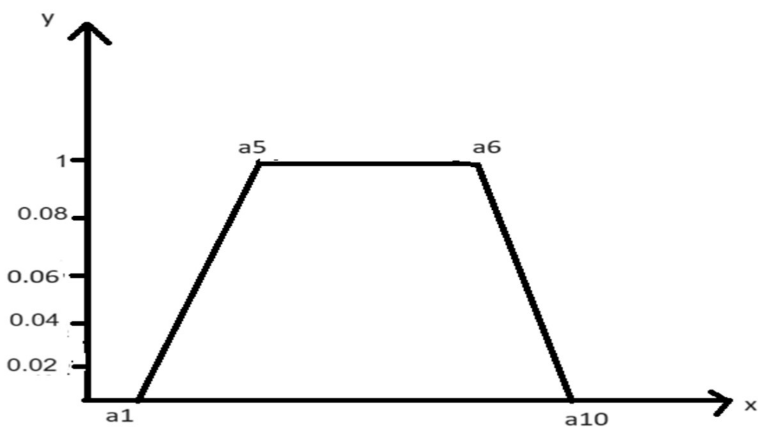

Fig 2 Graphical representation of Decagonal Fuzzy number

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$$\mu_{\widetilde{D2}}(x)=\begin{cases} 0 & x<a_1 \\ \frac{1}{50}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1\le x\le a_2 \\ \frac{1}{50}+\frac{1}{50}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2\le x\le a_3 \\ \frac{1}{25}+\frac{1}{50}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3\le x\le a_4 \\ \frac{3}{50}+\frac{47}{50}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4\le x\le a_5 \\ 1 & a_5\le x\le a_6 \\ 1-\frac{47}{50}\left(\frac{x-a_6}{a_7-a_6}\right) & a_6\le x\le a_7 \\ \frac{3}{50}-\frac{1}{25}\left(\frac{x-a_7}{a_8-a_7}\right) & a_7\le x\le a_8 \\ \frac{1}{25}-\frac{1}{25}\left(\frac{x-a_8}{a_9-a_8}\right) & a_8\le x\le a_9 \\ \frac{1}{50}-\frac{1}{50}\left(\frac{x-a_9}{a_{10}-a_9}\right) & a_9\le x\le a_{10} \\ 0 & \text{otherwise} \end{cases}$$

and ranking $as\ \Re(\widetilde{D2}) = \frac{a_1+0.02+0.04a_3+0.06a_4+a_5+a_6+0.06a_7+0.04+0.02+a_{10}}{4}$

and decagonal fuzzy number defined as $\mu_{\widetilde{D2}} = (L_1(u),\ M_1(v), N_1(w), O_2(t))$ for u ∈ [0, 0.02], v [0.02, 0.04], w [0.04, 0.06], t [0.06, 1] were,

(i) $L_1(u)$ is a bounded left continuous non decreasing function over [0, 0.02]
(ii) $M_1(u)$ is a bounded left continuous non decreasing function over [0.02, 0.04]
(iii) $N_1(w)$ is a bounded left continuous non decreasing function over[0.04, 0.06]
(iv) $O_1(t)$ is a bounded left continuous non decreasing function over [0.06, 1]
(v) $L_2(u)$ is a bounded continuous non increasing function over [1, 0.06]
(vi) $M_2(v)$ is a bounded continuous non increasing function over [0.06, 0.04]
(vii) $N_2(w)$ is a bounded continuous non increasing function over [0.04, 0.02]
$(viii)$ $O_2(t)$ is a bounded continuous non increasing function over [0.02, 0]

Definition 2.5. A fuzzy number $\mu_{\widetilde{D3}}$ is a decagonal fuzzy number denoted by $\mu_{\widetilde{D3}}$ $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ are real numbers and its membership function $\mu_{\widetilde{D3}}(x)$ is given below based on the intervals [0, 1].

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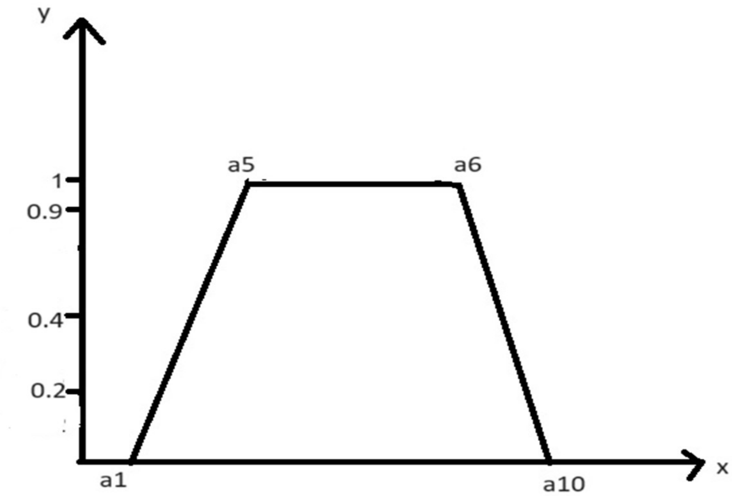

Fig 3 Graphical representation of Decagonal Fuzzy number

$$\mu_{\widetilde{D3}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{1}{5}\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ \frac{1}{5}+\frac{1}{5}\left(\frac{x-a_2}{a_3-a_2}\right) & a_2 \leq x \leq a_3 \\ \frac{2}{5}+\frac{1}{2}\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \\ \frac{9}{10}+\frac{1}{10}\left(\frac{x-a_4}{a_5-a_4}\right) & a_4 \leq x \leq a_5 \\ 1 & a_5 \leq x \leq a_6 \\ 1-\frac{1}{10}\left(\frac{x-a_6}{a_7-a_6}\right) & a_6 \leq x \leq a_7 \\ \frac{9}{10}-\frac{1}{2}\left(\frac{x-a_7}{a_8-a_7}\right) & a_7 \leq x \leq a_8 \\ \frac{2}{5}-\frac{2}{5}\left(\frac{x-a_8}{a_9-a_8}\right) & a_8 \leq x \leq a_9 \\ \frac{1}{5}-\frac{1}{5}\left(\frac{x-a_9}{a_{10}-a_9}\right) & a_9 \leq x \leq a_{10} \\ 0 & \text{otherwise} \end{cases}$$

and ranking function as $\Re(\widetilde{D3}) = \frac{a_1+0.2a_2+0.54+0.9+a_5+a_6+0.9a_7+0.4a_8+0.2a_9+a_{10}}{7}$ and decagonal fuzzy number defined as $\mu_{\widetilde{D3}} = (L_1(u), M_1(v), N_1(w), O_2(t))$ for u ∈ [0, 0.2], v [0.2, 0.4], w [0.4, 0.9], t [0.9, 1] were,

(i) $L_1(u)$ is a bounded left continuous non decreasing function over [0, 0.2]

(ii) $M_1(u)$ is a bounded left continuous non decreasing function over [0.2, 0.4]

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(iii) $N_1\,(w)$ is a bounded left continuous non decreasing function over [0.4, 0.9]
(iv) $O_1\,(t)$ is a bounded left continuous non decreasing function over [0.9, 1]
(v) $L_2\,(u)$ is a bounded continuous non increasing function over [1, 0.9]
(vi) $M_2\,(v)$ is a bounded continuous non increasing function over [0.9, 0.4]
(vii) $N_2(w)$ is a bounded continuous non increasing function over [0.4, 0.2]
(viii) $O_2\,(t)$ is a bounded continuous non increasing function over [0.2, 1]

Definition 2.7. A positive decagonal fuzzy number $\mu_{\widetilde{D}}$ is denoted as $\mu_{\widetilde{D}}$= $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ where all a_i's > 0 for all i = 1 to10.

Definition 2.8. A negative decagonal fuzzy number $\mu_{\widetilde{D}}$ is denoted as $\mu_{\widetilde{D}}$= $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})$ where all a_i's < 0 for all i = 1, 2, 3, 4, 5, 6,7,8,9,10.

Example 2.9. A tourist buses visit two places in different vehicles. Formulate to minimum distance using univariate search method.

f = (3.2, 3.3, 3.5, 3.6, 3.7, 3.8,4.0,4.7, 5.1, 5.3) $\tilde{x}_1^2$ + (1.1, 1.2, 1.5, 2.0, 2.2, 2.3,2.4,2.7,2.8) $\tilde{x}_2^2$

Solution.

Let length $\in= 0.1$ and given $X_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and using the ranking function $\Re(\widetilde{D1})$ converted into crisp value i.e. $f = 4{x_1}^2 + 2{x_2}^2$

Assume search direction $s_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, then $f_1 = 48$

$f^+ = (X_1 + \in s_1) = f\begin{bmatrix} 2.2 \\ 4 \end{bmatrix} = 51 > f_1$,

$f^- = (X_1 - \in s_1) = f\begin{bmatrix} 1.8 \\ 4 \end{bmatrix} = 44.96 < f_1$.

Thus, $-s_1$ is the correct direction for minimum distance f from X_1.

For minimizing, $f(X_1 - \mu_1 s_1)$, $\frac{df}{d\mu_1} = 0$, $\mu_1 = 1$

Let $X_2 = X_1 - \mu_1 s_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$, Choose second direction $s_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then $f_2 = 32$

$f^+ = (X_2 + \in s_2) = f\begin{bmatrix} 0 \\ 4.1 \end{bmatrix} = 33.62 > f_2$,

$f^- = (X_2 - \in s_2) = f\begin{bmatrix} 0 \\ 3.9 \end{bmatrix} = 30.42 < f_2$.

Thus, $-s_2$ is the correct direction for minimum distance f from X_2.

For minimizing, $f(X_2 - \mu_2 s_2)$, $\frac{df}{d\mu_2} = 0$, $\mu_2 = 4$

Let $X_3 = X_2 - \mu_2 s_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, Choose again search direction $s_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, then $f_3 = 0$

$f^+ = (X_3 + \in s_1) = f\begin{bmatrix} 0.1 \\ 0 \end{bmatrix} = 0.01 > f_3$,

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$f^{-} = (X_3 - \in s_1) = f\begin{bmatrix} -0.1 \\ 0 \end{bmatrix} = 0.01 > f_3$. We conclude that $X_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ can be taken as minimum point. Hence, we get X values are zero, reached destination with minimum distance.

Similarly, to prove the remaining two ranking functions $\mathfrak{R}(\widetilde{D2})$ and $\mathfrak{R}(\widetilde{D3})$, we get best route to get our destination.

3. Conclusion

The fuzzy optimal solution of Decagonal fuzzy numbers turn into crisp numbers using a new Ranking function is solved utilizing a new approach presented in this research. Using the three ranking functions mentioned above, we have the same outcomes. Calculating the decagonal weights of criteria can be done with a sensible and efficient ranking algorithm. Consequently, solving optimization strategies is simpler. This technique makes it simple to navigate from place to place turn-by-turn and to determine the most efficient way to get there.

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