FINDING CRITICAL PATHS IN A FUZZY PROJECT NETWORK WITH OCTAGONAL FUZZY NUMBERS

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Abstract:-
Production-based calculations of activity duration are possible, but sometimes the results of this approach are estimations that are too general and imprecise. To get around this problem, we can use fuzzy numbers to define the activity periods. As a result, the activity durations in this paper are represented by octagonal fuzzy numbers (a, b, c, d, e, f, g, h).
A clear numerical example is provided to showcase the superiority of the proposed algorithm and method.

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1. Introduction:-

The current era is marked by intense competition and rapid development across multiple fields. This environment not only fosters innovation but also presents challenges and opportunities as industries and individuals strive to adapt and excel. To improve the accuracy of project activity duration estimates, it is essential to consider these factors during the planning phase. Employing experienced project managers, investing in training, using modern equipment, conducting thorough risk assessments, and maintaining effective communication can help mitigate these issues. Technological advancements, globalization, and the rapid exchange of information have created a highly competitive environment in numerous sectors, fostering innovation and progress. Unrealistic project activity duration estimates can be caused by a variety of factors, including insufficient expertise, a shortage of skilled labour, out-dated equipment, unstable markets, inclement weather, and cash flow. The Critical Path Method (CPM) is an essential technique for planning and managing construction projects. The time of each task must be accurately stated in order for the Critical Path Method to be used correctly. However, it is frequently impossible to determine how long an action will take in the real world. Modern corporate environments are highly variable, thus using advanced technical planning, monitoring, and controlling strategies is essential. One of the methods in this study is the fuzzy critical path method.

Inadequate and weak project planning leads to goal failure and project completion delays. Modern planning approaches are necessary for large projects in order to conquer the complicated nature and interruption of activity. Inadequate and weak project planning leads to goal failure and delays in project completion. Complex projects need to apply modern planning strategies to deal with the complexity and interference of activities. The concept of fuzzy logic is an appropriate choice for situations where the length of an activity is unclear.
Anusuya.V. and Balasowandari.P. derived the fuzzy critical path from type-2 trapezoidal fuzzy numbers in an acyclic project network by using magnitude measure [1]. Beaula and Vijaya used a trapezoidal fuzzy number-based linear programming technique to find the critical path from many critical paths [3]. The shortest path problem was first studied by Dubois and Prade [4]. It is a common problem in practice and is simple to solve well. Using $\alpha -$ cuts, Radhakrishnan.S and Saikeerthana.D [7] transformed the fuzzy parameters (trapezoidal and triangle fuzzy numbers) into intervals. T. Yogashanti [9] and others proposed the intuitionistic fuzzy variation of the critical route technique to handle networking problems requiring uncertain activity durations. D. Stephen Dinagar and Abirami [8] introduced a novel method for identifying the critical path using the minimal path length while working with interval valued fuzzy networks. Zadeh [10] introduced the concept of fuzzy sets in 1965, and it is important to address circumstances such as these. In order to overcome the ambiguity associated with calculating activity durations, Awss Hatim Mahmoud [2] presented a novel approach for determining activity durations in the face of uncertainty. To find the fuzzy critical path and length, Elizebeth.S and Sujatha.L [5] proposed novel ranking algorithms for triangular fuzzy numbers. Rameshan N. and Stephen Dinagar, D. proposed a novel method [6] for calculating the fuzzy critical route utilizing the Octagonal fuzzy number's activity time.

2. Preliminaries

Definition 2.1
A fuzzy set $\widetilde{U}$ in X is a set of order pair defined by, $\widetilde{U}(x) = \{x, \mu_{\widetilde{U}}(x), x \in X, \mu_{\widetilde{U}}(x) \in [0,1]\}$, where $\mu_{\widetilde{U}}(x)$ is a membership function.

Definition 2.2
A fuzzy set $\widetilde{U}$ defined on a set of real number R is said to be a fuzzy number, if its membership function $\widetilde{U}$: R $\rightarrow [0,1]$ has the following characteristic

i) $\widetilde{U}$ is convex, ie $\widetilde{U}\{\lambda x_1 + (1-\lambda)x_2\} \geq min\{\widetilde{U}(x_1), \widetilde{U}(x_2)\}$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$
ii) $\widetilde{U}$ is normal, i.e. there exist an $x \in R \ni \widetilde{U}(x) = 1$

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iii) $\widetilde{U}$ is a piecewise continuous.

Definition 2.3

A fuzzy number $\widetilde{U}$ is octagonal fuzzy number denoted by $\widetilde{U} = (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8)$, the membership function is

$$\mu_{\tilde{F}}(x) = \begin{cases} 0 & , x \le u_1 \\ \lambda\left(\frac{x-u_1}{u_2-u_1}\right) & , u_1 \le x \le u_2 \\ \lambda & , u_2 \le x \le u_3 \\ \lambda + (1-\lambda)\left(\frac{x-u_3}{u_4-u_3}\right) & , u_3 \le x \le u_4 \\ 1 & , u_4 \le x \le u_5 \\ \lambda + (1-\lambda)\left(\frac{u_6-x}{u_6-u_5}\right) & , u_5 \le x \le u_6 \\ \lambda & , u_6 \le x \le u_7 \\ \lambda\left(\frac{u_8-x}{u_8-u_7}\right) & , u_7 \le x \le u_8 \\ 0 & , x \ge u_8 \end{cases}$$

, where $0 < \lambda < 1$

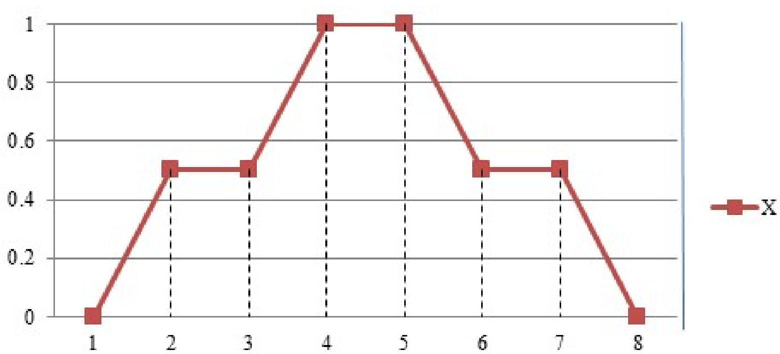

Fig 2.1: Graphical Representation of OCFN

3. Fuzzy Delphi Method

The Fuzzy Delphi Method (FDM) is indeed a classical forecasting method in management science. It combines the traditional Delphi method, which relies on the structured communication of a panel of experts to achieve consensus, with fuzzy logic to handle the inherent uncertainty and vagueness in expert opinions. Here's an overview of how the Fuzzy Delphi Method works and its applications in management science:

- An overview of the project's activity is sent to eight competent project management specialists.

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- For every project activity, each expert submits eight times, which represents an octagonal fuzzy number$\widetilde{U} = (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8)$; experts are unaware of each other's numbers of experts and time estimates.

- The average is calculated $A = (au_1, au_2, au_3, au_4, au_5, au_6, au_7, au_8) =$
$$\left(\frac{1}{n}\sum_{i=1}^{n} u_{1_i}, \frac{1}{n}\sum_{i=1}^{n} u_{2_i}, \frac{1}{n}\sum_{i=1}^{n} u_{3_i}, \frac{1}{n}\sum_{i=1}^{n} u_{4_i}, \frac{1}{n}\sum_{i=1}^{n} u_{5_i}, \frac{1}{n}\sum_{i=1}^{n} u_{6_i}, \frac{1}{n}\sum_{i=1}^{n} u_{7_i}, \frac{1}{n}\sum_{i=1}^{n} u_{8_i}\right) \quad (1)$$

  - Equation (2) shows how the difference (D) between the mean (M) and $\widetilde{U}$ is calculated for each expert
$$D_i = (M - E_i) = \left(mu_1 - u_{1_i}, mu_2 - u_{2_i} mu_3 - u_{3_i} mu_4 - u_{4_i}\right)$$
  - Each expert receives the deviation (D_i) and is required to submit an opportunity to take trapezoidal fuzzy number.
  - The procedure is repeated until two succeeding means, A1, A2, and A3, are sufficiently close to each other.

4. Illustration

Table 4.1 Fuzzy Octagonal Durations

Event	Activities	Description	Fuzzy Durations
1-2	A	Identify the Maintenance requirements	(14,15,16,17,18,19,20,21)
2-3	B	Arranging the task in logical order	(13,15,17,19,20,22,23,24)
3-5	C	Skills requirement	(14,16,17,18,20,21,23,24)
2-4	D	Resource availability	(4,6,7,8,10,11,13,15)
4-5	E	Specifying the tools	(11,12,13,14,16,17,18,19)
4-7	F	Cleaning	(18,20,22,24,26,28,30,32)
5-6	G	Part Replacement	(15,17,19,21,23,25,27,29)
7-8	H	Testing	(3,5,6,6,7,8,9,11)
6-8	I	Inspection	(6,7,8,9,10,11,12,13,14)
8-9	J	Documentation	(24,25,26,27,28,29,30,31)

Planning and sequencing activities for a maintenance job involves several steps to ensure the job is completed efficiently and effectively. This process typically includes defining the scope of work, breaking down the tasks, determining the sequence of activities, scheduling, and monitoring progress.
Ten project activities and all project-related data were selected by the researcher for the investigation. Eight qualified specialists in the field of maintenance project management were selected to submit eight durations for each of the 10 activities, together with all project-related data. The Delphi method is used by the researcher to analyse the eight expert durations. The stages and sequence needed to complete a maintenance task are listed in the table.

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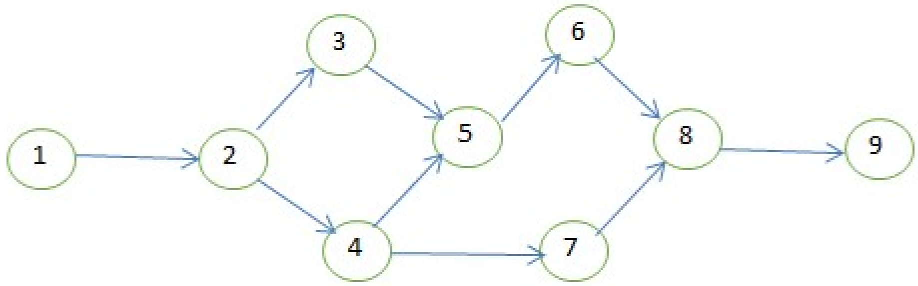

4.1 Network

Project Network Calculations

Table 4.2: **Fuzzy Earliest Start and Finish**

Event	Fuzzy Durations	Fuzzy Earliest Start	Fuzzy Earliest Finish
1-2	(14,15,16,17,18,19,20,21)	(0,0,0,0,0,0,0,0)	(14,15,16,17,18,19,20,21)
2-3	(13,15,17,19,20,22,23,24)	(14,15,16,17,18,19,20,21)	(27,30,31,36,38,41,43,45)
3-5	(14,16,17,18,20,21,23,24)	(27,30,31,36,38,41,43,45)	(41,46,47,56,58,62,66,69)
2-4	(4,6,7,8,10,11,13,15)	(14,15,16,17,18,19,20,21)	(20,21,23,25,28,30,33,36)
4-5	(11,12,13,14,16,17,18,19)	(20,21,23,25,28,30,33,36)	(31,33,36,39,44,47,51,55)
4-7	(18,20,22,24,26,28,30,32)	(20,21,23,25,28,30,33,36)	(38,41,45,49,54,58,63,68)
5-6	(15,17,19,21,23,25,27,29)	(41,46,47,56,58,62,66,69)	(56,63,66,77,81,87,83,98)
7-8	(3,5,6,7,8,9,11,12)	(38,41,45,49,54,58,63,68)	(41,46,51,54,62,67,74,80)
6-8	(6,7,8,9,10,11,12,13)	(56,63,66,77,81,87,83,98)	(62,70,74,87,92,98,95,111)
8-9	(24,25,26,27,28,29,30,31)	(62,70,74,87,92,98,95,111)	(86,95,100,114,120,127,125,142)

Table 4.3: **Fuzzy Latest Start and Finish**

Event	Fuzzy Durations	Fuzzy Latest Start	Fuzzy Latest Finish

1-2	(14,15,16,17,18,19,20,21)	(-56,-40,-27,-5,8,24,30,57)	(-35,-20,-8,13,24,40,45,71)
2-3	(13,15,17,19,20,22,23,24)	(-35,-20,-8,13,24,40,45,71)	(-11,3,14,33,43,57,60,84)
3-5	(14,16,17,18,20,21,23,24)	(-11,3,14,33,43,57,60,84)	(13,26,35,53,61,74,76,90)
2-4	(4,6,7,8,10,11,13,15)	(-21,-5,7,27,39,52,58,75)	(-6,8,18,37,47,61,64,79)
4-5	(11,12,13,14,16,17,18,19)	(-6,8,18,37,47,61,64,79)	(13,26,35,53,61,74,76,90)
4-7	(18,20,22,24,26,28,30,32)	(11,24,34,52,62,73,75,97)	(43,54,62,78,86,95,95 ,115)
5-6	(15,17,19,21,23,25,27,29)	(13,26,35,53,61,74,76,90)	(42,53,60,76,82,93,93,105)
7-8	(3,5,6,7,8,9,11,12)	(43,54,62,78,86,95,95 ,115)	(55,65,71,86,93,101,100,118)
6-8	(6,7,8,9,10,11,12,13)	(42,53,60,76,82,93,93,105)	(55,65,71,86,93,101,100,118)
8-9	(24,25,26,27,28,29,30,31)	(55,65,71,86,93,101,100,118)	(86,95,100,114,120,127,125,142)

Table 4.4: **Total Float**

Event	Fuzzy Latest Start	Fuzzy Earliest Start	Fuzzy Total Float
1-2	(-56,-40,-27,-5,8,24,30,57)	(0,0,0,0,0,0,0,0)	**(-56,-40,-27,-5,8,24,30,57)**
2-3	(-35,-20,-8,13,24,40,45,71)	(14,15,16,17,18,19,20,21)	**(-56,-40,-27,-5,7,24,30,57)**
3-5	(-11,3,14,33,43,57,60,84)	(27,30,31,36,38,41,43,45)	**(-56,-40,-27,-5,7,26,30,57)**
2-4	(-21,-5,7,27,39,52,58,75)	(14,15,16,17,18,19,20,21)	(-42,-25,-12,9,22,36,43,61)
4-5	(-6,8,18,37,47,61,64,79)	(20,21,23,25,28,30,33,36)	(-42,-25,-12,9,22,38,31,59)
4-7	(11,24,34,52,62,73,75,97)	(20,21,23,25,28,30,33,36)	(-25,-9,4,24,37,50,54,77)
5-6	(13,26,35,53,61,74,76,90)	(41,46,47,56,58,62,66,69)	**(-56,-40,-27,-5,5,27,30,49)**
7-8	(43,54,62,78,86,95,95 ,115)	(38,41,45,49,54,58,63,68)	(-25,-9,4,24,37,50,54,77)

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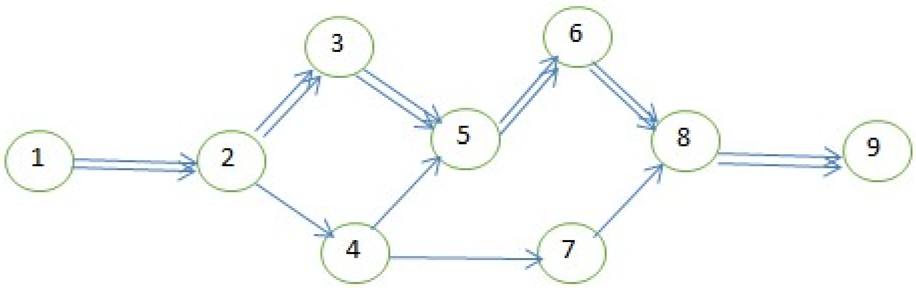
6-8	(42,53,60,76,82,93,93,105)	(56,63,66,77,81,87,83,98)	**(-56,-30,-27,-5,5,27,30,49)**
8-9	(55,65,71,86,93,101,100,118)	(62,70,74,87,92,98,95,111)	**(-56,-30,-27,-6,6,27,30,56)**



4.2 Critical Path

The critical path is 1-2-3-5-6-8

5. Conclusion:-

In this research, we introduce a new critical path strategy designed to determine the optimal time for solving a fuzzy networking problem. This approach leverages octagonal fuzzy numbers and maintains the problem's inherent fuzziness, avoiding the need to convert it into a conventional networking problem. Maintenance activities indeed involve a high degree of variability due to factors such as the type of equipment, operating conditions, and the specific maintenance tasks required. To handle this complexity, advanced technical methods like fuzzy sets and fuzzy logic can be highly beneficial. Maintenance managers can use fuzzy logic systems to evaluate the urgency and priority of different maintenance tasks, helping to allocate resources more effectively. This Fuzzy critical path method allows for more flexible and realistic scheduling.

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