INTRODUCTION ABOUT BIPOLAR VALUED VAGUE SUBNEARRINGS OF A NEARRING

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ABSTRACT.
In this paper, bipolar valued vague subnearring of a nearring is introduced and some properties are discussed. Here union and intersection are applied in bipolar valued vague subnearring of a nearing. It is proved that product of two bipolar valued vague subnearring of a nearing is a bipolar valued vague subnearring. Some properties of bipolar valued vague subnearring of a nearring are discussed.

KEY WORDS. Fuzzy subset, vague subset, bipolar valued fuzzy subset, bipolar valued vague subset, bipolar valued vague subnearring, intersection and product.

INTRODUCTION:
In 1965, Zadeh [13] introduced the notion of a fuzzy subset of a Universal set. Vague set is an extension of fuzzy set and it is appeared as a unique case of context dependent fuzzy sets. The vague set was introduced by W.L.Gau and D.J.Buehrer [5]. W.R.Zhang [14, 15] introduced an extension of fuzzy sets named bipolar valued fuzzy sets in 1994 and bipolar valued fuzzy set was developed by Lee [6, 7]. Fuzzy subgroup was introduced by Azriel Rosenfeld [3]. RanjitBiswas [9] introduced the vague groups. Cicily Flora. S and Arockiarani.I [4] have introduced a new class of generalized bipolar vague sets. Anitha.M.S., et.al.[1] defined as bipolar valued fuzzy subgroups of a group. Sheena. K. P and K.Uma Devi [10] have introduced the bipolar valued fuzzy subbigroup of a bigroup. Shanthi.V.K and G.Shyamala[11] have introduced the bipolar valued multi fuzzy subgroups of a group. Yasodara.S, KE. Sathappan [12] defined the bipolar valued multi fuzzy subsemirings of a semiring. Bipolar valued multi fuzzy subnearring of a nearing has been introduced by S.Muthukumaran and B.Anandh [8]. Anitha.K., et.al.[2] defined as bipolar valued vague subsemirings of a semiring. Here, the concept of bipolar valued vague subnearring of a nearring is introduced and estaiblished some results.

1.PRELIMINARIES.

Definition 1.1. [13] Let X be any nonempty set. A mapping M : X → [0, 1] is called a fuzzy subset of X.

Definition 1.2. [5] A vague set A in the universe of discourse U is a pair [t_A, $1-f_A$],where t_A : U→ [0, 1] and f_A : U→ [0, 1] are mappings, they are called truth membership function and false membership function respectively. Here t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and f_A(x) is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \leq 1$, for all x∈U.

Definition 1.3. [5] The interval [$t_A(x)$, $1-f_A(x)$] is called the vague value of x in A and it is denoted by $V_A(x)$, i.e., $V_A(x)$ = [$t_A(x)$, $1-f_A(x)$].

Example 1.4. A = { < a, [0.1, 0.2] >, < b, [0.3, 0.4] >, < c, [0.5, 0.6] >} is a vague subset of X = {a, b, c }.

Definition 1.5. [14] A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where A^+ : X→ [0, 1] and A^- : X→ [−1, 0]. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6. A = { < a, 0.2, −0.3 >, < b, 0.4, −0.5 >, < c, 0.6, −0.7 >} is a bipolar valued fuzzy subset of X = {a, b, c }.

Definition 1.7. [4] A bipolar valued vague subset A in X is defined as an object of the form A = { ⟨ x, [$t_A^+(x), 1-f_A^+(x)$], [$-1-f_A^-(x), t_A^-(x)$] ⟩ / x∈X }, where t_A^+: X→ [0, 1], f_A^+ : X→ [0, 1], t_A^- : X→ [−1, 0] and f_A^- : X→ [−1, 0] are mapping such that $t_A(x) + f_A(x) \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive interval membership degree [$t_A^+(x), 1-f_A^+(x)$] denotes the satisfaction region of an element x to the property corresponding to a bipolar valued vague subset A and the negative interval membership degree [$-1-f_A^-(x), t_A^-(x)$] denotes the satisfaction region of an element x to some implicit counter-property corresponding to a bipolar valued vague subset A. Bipolar valued vague subset A is denoted as A = { ⟨ x, $V_A^+(x)$, $V_A^-(x)$ ⟩ / x∈X }, where $V_A^+(x)$,= [$t_A^+(x), 1-f_A^+(x)$] and $V_A^-(x)$ = [$-1-f_A^-(x), t_A^-(x)$].

Note that. [0] = [0, 0], [1] = [1, 1] and [−1] = [−1, −1].

Example 1.8. [A] = { < a, [0.5, 0,8], [−0.4, −0.1] >, < b, [0.22, 0.54], [−0.7, −0.2] >, < c, [0.11, 0.5], [−0.8, −0.5] > } is a bipolar valued vague subset of X = {a, b, c }.

Definition 1.9. [4] Let A = ⟨V_A^+, V_A^-⟩ and B = ⟨V_B^+, V_B^-⟩ be two bipolar valued vague subsets of a set X. We define the following relations and operations:

(i) A ⊂ B if and only if $V_A^+(u) \leq V_B^+(u)$ and $V_A^-(u) \geq V_B^-(u)$, ∀ u∈X.

(ii) A = B if and only if $V_A^+(u) = V_B^+(u)$ and $V_A^-(u) = V_B^-(u)$, ∀ u∈X.

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(iii) A∩B = { ⟨ u, rmin (V_A^+ (u), V_B^+ (u)), rmax (V_A^- (u), V_B^- (u)) ⟩ / u∈X }.

(iv) A∪B = { ⟨ u, rmax (V_A^+ (u), V_B^+ (u)), rmin (V_A^- (u), V_B^- (u)) ⟩ / u∈X }. Here rmin (V_A^+ (u), V_B^+ (u)) = [min { $t_A^+(x)$, $t_B^+(x)$ }, min { $1-f_A^+(x)$, $1-f_B^+(x)$ }], rmax (V_A^+ (u), V_B^+ (u)) = [max { $t_A^+(x)$, $t_B^+(x)$ }, max { $1-f_A^+(x)$, $1-f_B^+(x)$ }], rmin (V_A^- (u), V_B^- (u)) = [min { $-1-f_A^-(x)$, $-1-f_B^-(x)$ }, min { $t_A^-(x)$, $t_B^-(x)$ }], rmax (V_A^- (u), V_B^- (u)) = [max { $-1-f_A^-(x)$, $-1-f_B^-(x)$ }, max { $t_A^-(x)$, $t_B^-(x)$ }].

Definition 1.10. Let R be a nearring. A bipolar valued vague subset A of R is said to be a bipolar valued vague subnearring of R (BVVSNR) if the following conditions are satisfied,

(i) V_A^+ (x−y) ≥ rmin{ V_A^+ (x), V_A^+ (y) }

(ii) V_A^+ (xy) ≥ rmin{ V_A^+ (x), V_A^+ (y) }

(iii) V_A^- (x−y) ≤ rmax{ V_A^- (x), V_A^- (y) }

(iv) V_A^- (xy) ≤ rmax{ V_A^- (x), V_A^- (y) } for all x and y in R.

Example 1.11. Let R = Z_3 = { 0, 1, 2 } be a nearring with respect to the ordinary addition and multiplication. Then A = { < 0, [0.6, 0.8], [− 0.9, − 0.6] >, < 1, [0.5, 0.7], [− 0.8, − 0.5] >, < 2, [0.5, 0.7], [− 0.8, − 0.5] > } is a bipolar valued vague subnearring of R.

Definition 1.12. Let A = ⟨ V_A^+ , V_A^- ⟩ and B = ⟨ V_B^+ , V_B^- ⟩ be any two bipolar valued vague subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as A×B = { ⟨ (x, y), $V_{A\times B}^+$ (x, y), $V_{A\times B}^-$ (x, y) ⟩ / for all x in G and y in H } where $V_{A\times B}^+$ (x, y) = rmin { V_A^+ (x), V_B^+ (y) } and $V_{A\times B}^-$ (x, y) = rmax { V_A^- (x), V_B^- (y) } for all x in G and y in H.

2. THEOREMS.

Theorem 2.1. If A = ⟨ V_A^+ , V_A^- ⟩ and B = ⟨ V_B^+ , V_B^- ⟩ are two bipolar valued vague subnearrings of a nearring R, then their intersection A∩B is a bipolar valued vague subnearring of R.

Proof. Let C = A∩B and let x, y in R. Now V_C^+ (x−y) = rmin { V_A^+ (x−y), V_B^+ (x−y) } ≥ rmin { rmin { V_A^+ (x), V_A^+ (y) }, rmin { V_B^+ (x), V_B^+ (y) } } ≥ rmin { rmin { V_A^+ (x), V_B^+ (x) }, rmin { V_A^+ (y), V_B^+ (y)} } = rmin { V_C^+ (x), V_C^+ (y) }. Therefore V_C^+ (x−y) ≥ rmin{ V_C^+ (x), V_C^+ (y) }, for all x, y in R. And V_C^+ (xy) = rmin { V_A^+ (xy), V_B^+ (xy) } ≥ rmin { rmin{ V_A^+ (x), V_A^+ (y) }, rmin { V_B^+ (x), V_B^+ (y) }} ≥ rmin { rmin { V_A^+ (x), V_B^+ (x) }, rmin { V_A^+ (y), V_B^+ (y) } } = rmin { V_C^+ (x), V_C^+ (y) }. Therefore V_C^+ (xy) ≥ rmin { V_C^+ (x), V_C^+ (y) }, for all x, y in R. Also V_C^- (x−y) = rmax { V_A^- (x−y), V_B^- (x−y) } ≤ rmax { rmax { V_A^- (x), V_A^- (y) }, rmax{ V_B^- (x), V_B^- (y) }} ≤ rmax { rmax { V_A^- (x), V_B^- (x) }, rmax{ V_A^- (y), V_B^- (y) } } = rmax { V_C^- (x), V_C^- (y) }. Therefore V_C^- (x−y) ≤ rmax { V_C^- (x),

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V_C^- (y) }, for all x, y in R. And V_C^- (xy) = rmax { V_A^- (xy), V_B^- (xy) } ≤ rmax { rmax { V_A^- (x), V_A^- (y) }, rmax { V_B^- (x), V_B^- (y) } } ≤ rmax { rmax { V_A^- (x), V_B^- (x) }, rmax { V_A^- (y), V_B^- (y) } }= rmax { V_C^- (x), V_C^- (y) }. Therefore V_C^- (xy) ≤ rmax { V_C^- (x), V_C^- (y) }, for all x, y in R. Hence A∩B is a bipolar valued vague subnearring of R.

Theorem 2.2. The intersection of a family of bipolar valued vague subnearrings of a nearring R is a bipolar valued vague subnearring of R.

Proof. The proof follows from the Theorem 2.1.

Theorem 2.3. If A = ⟨V_A^+, V_A^-⟩ and B = ⟨V_B^+, V_B^-⟩ are two bipolar valued vague subnearrings of a nearring R, then their union A∪B need not be a bipolar valued vague subnearring of R.

Proof. Since union of any two nearrings need not be a nearing, so union A∪B need not be a bipolar valued vague subnearring of R.

Theorem 2.4. If A = ⟨V_A^+, V_A^-⟩ and B = ⟨V_B^+, V_B^-⟩ are any two bipolar valued vague subnearrings of the nearrings R_1 and R_2 respectively, then A×B = ⟨ $V_{A\times B}^+$, $V_{A\times B}^-$ ⟩ is a bipolar valued vague subnearring of $R_1\times R_2$.

Proof. Let x_1, x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1\times R_2$. Now, $V_{A\times B}^+$ [(x_1, y_1) – (x_2, y_2)] = $V_{A\times B}^+$ (x_1-x_2, y_1-y_2) = rmin { V_A^+ (x_1-x_2), V_B^+ (y_1-y_2) } ≥ rmin { rmin{ V_A^+ (x_1), V_A^+ (x_2) }, rmin { V_B^+ (y_1), V_B^+ (y_2) }} = rmin { rmin { V_A^+ (x_1), V_B^+ (y_1) }, rmin { V_A^+ (x_2), V_B^+ (y_2) } } = rmin { $V_{A\times B}^+$ (x_1, y_1), $V_{A\times B}^+$ (x_2, y_2) }. Therefore $V_{A\times B}^+$ [(x_1, y_1) –(x_2, y_2)] ≥ rmin { $V_{A\times B}^+$ (x_1, y_1), $V_{A\times B}^+$ (x_2, y_2) }. And $V_{A\times B}^+$ [(x_1, y_1)(x_2, y_2)] = $V_{A\times B}^+$ (x_1x_2, y_1y_2) = rmin { V_A^+ (x_1x_2), V_B^+ (y_1y_2) } ≥ rmin { rmin { V_A^+ (x_1), V_A^+ (x_2) }, rmin { V_B^+ (y_1), V_B^+ (y_2) } } = rmin { rmin { V_A^+ (x_1), V_B^+ (y_1) }, rmin { V_A^+ (x_2), V_B^+ (y_2) } } = rmin { $V_{A\times B}^+$ (x_1, y_1), $V_{A\times B}^+$ (x_2, y_2) }. Therefore $V_{A\times B}^+$ [(x_1, y_1)(x_2, y_2)] ≥ rmin{ $V_{A\times B}^+$ (x_1, y_1), $V_{A\times B}^+$ (x_2, y_2) }. Also $V_{A\times B}^-$ [(x_1, y_1) – (x_2, y_2)] = $V_{A\times B}^-$ (x_1-x_2, y_1-y_2) = rmax { V_A^- (x_1-x_2), V_B^- (y_1-y_2) } ≤ rmax { rmax { V_A^- (x_1), V_A^- (x_2) }, rmax { V_B^- (y_1), V_B^- (y_2) }} = rmax { rmax { V_A^- (x_1), V_B^- (y_1) }, rmax { V_A^- (x_2), V_B^- (y_2) }} = rmax { $V_{A\times B}^-$ (x_1, y_1), $V_{A\times B}^-$ (x_2, y_2) }. Therefore $V_{A\times B}^-$ [(x_1, y_1) –(x_2, y_2)] ≤ rmax { $V_{A\times B}^-$ (x_1, y_1), $V_{A\times B}^-$ (x_2, y_2) }. And $V_{A\times B}^-$ [(x_1, y_1)(x_2, y_2)] = $V_{A\times B}^-$ (x_1x_2, y_1y_2) = rmax { V_A^- (x_1x_2), V_B^- (y_1y_2) } ≤ rmax { rmax { V_A^- (x_1), V_A^- (x_2) }, rmax { V_B^- (y_1), V_B^- (y_2) } } = rmax { rmax { V_A^- (x_1), V_B^- (y_1) }, rmax { V_A^- (x_2), V_B^- (y_2) } } = rmax { $V_{A\times B}^-$ (x_1, y_1), $V_{A\times B}^-$ (x_2, y_2) }. Therefore $V_{A\times B}^-$ [(x_1, y_1)(x_2, y_2)] ≤ rmax{ $V_{A\times B}^-$ (x_1, y_1), $V_{A\times B}^-$ (x_2, y_2) }. Hence A×B is a bipolar valued vague subnearring of $R_1\times R_2$.

Theorem 2.5. If A = ⟨V_A^+, V_A^-⟩, B = ⟨V_B^+, V_B^-⟩, … K = ⟨V_K^+, V_K^-⟩ are bipolar valued vague

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subnearrings of the nearrings R_A, R_B, …, R_K respectively, then A×B×…×K = $\langle V^{+}_{A\times B\times\ldots\times K}, V^{-}_{A\times B\times\ldots\times K} \rangle$ is a bipolar valued vague subnearring of $R_A\times R_B\times\ldots\times R_K$.

Proof. Let $(a_1, b_1, \ldots, k_1)$ and $(a_2, b_2, \ldots, k_2)$ are in $R_A\times R_B\times\ldots\times R_K$. Now, $V^{+}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1) - (a_2, b_2, \ldots, k_2)] = V^{+}_{A\times B\times\ldots\times K}(a_1-a_2, b_1-b_2, \ldots, k_1-k_2) = \text{rmin}\{V^{+}_{A}(a_1-a_2), V^{+}_{B}(b_1-b_2), \ldots, V^{+}_{K}(k_1-k_2)\} \geq \text{rmin}\{\text{rmin}\{V^{+}_{A}(a_1), V^{+}_{A}(a_2)\}, \text{rmin}\{V^{+}_{B}(b_1), V^{+}_{B}(b_2)\}, \ldots, \text{rmin}\{V^{+}_{K}(k_1), V^{+}_{K}(k_2)\}\} = \text{rmin}\{\text{rmin}\{V^{+}_{A}(a_1), V^{+}_{B}(b_1), \ldots, V^{+}_{K}(k_1)\}, \text{rmin}\{V^{+}_{A}(a_2), V^{+}_{B}(b_2), \ldots, V^{+}_{K}(k_2)\}\} = \text{rmin}\{V^{+}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{+}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. Therefore $V^{+}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1) - (a_2, b_2, \ldots, k_2)] \geq \text{rmin}\{V^{+}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{+}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. And $V^{+}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1)(a_2, b_2, \ldots, k_2)] = V^{+}_{A\times B\times\ldots\times K}(a_1a_2, b_1b_2, \ldots, k_1k_2) = \text{rmin}\{V^{+}_{A}(a_1a_2), V^{+}_{B}(b_1b_2), \ldots, V^{+}_{K}(k_1k_2)\} \geq \text{rmin}\{\text{rmin}\{V^{+}_{A}(a_1), V^{+}_{A}(a_2)\}, \text{rmin}\{V^{+}_{B}(b_1), V^{+}_{B}(b_2)\}, \ldots, \text{rmin}\{V^{+}_{K}(k_1), V^{+}_{K}(k_2)\}\} = \text{rmin}\{\text{rmin}\{V^{+}_{A}(a_1), V^{+}_{B}(b_1), \ldots, V^{+}_{K}(k_1)\}, \text{rmin}\{V^{+}_{A}(a_2), V^{+}_{B}(b_2), \ldots, V^{+}_{K}(k_2)\}\} = \text{rmin}\{V^{+}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{+}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. Therefore $V^{+}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1)(a_2, b_2, \ldots, k_2)] \geq \text{rmin}\{V^{+}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{+}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. Also $V^{-}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1) - (a_2, b_2, \ldots, k_2)] = V^{-}_{A\times B\times\ldots\times K}(a_1-a_2, b_1-b_2, \ldots, k_1-k_2) = \text{rmax}\{V^{-}_{A}(a_1-a_2), V^{-}_{B}(b_1-b_2), \ldots, V^{-}_{K}(k_1-k_2)\} \leq \text{rmax}\{\text{rmax}\{V^{-}_{A}(a_1), V^{-}_{A}(a_2)\}, \text{rmax}\{V^{-}_{B}(b_1), V^{-}_{B}(b_2)\}, \ldots, \text{rmax}\{V^{-}_{K}(k_1), V^{-}_{K}(k_2)\}\} = \text{rmax}\{\text{rmax}\{V^{-}_{A}(a_1), V^{-}_{B}(b_1), \ldots, V^{-}_{K}(k_1)\}, \text{rmax}\{V^{-}_{A}(a_2), V^{-}_{B}(b_2), \ldots, V^{-}_{K}(k_2)\}\} = \text{rmax}\{V^{-}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{-}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. Therefore $V^{-}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1) - (a_2, b_2, \ldots, k_2)] \leq \text{rmax}\{V^{-}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{-}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. And $V^{-}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1)(a_2, b_2, \ldots, k_2)] = V^{-}_{A\times B\times\ldots\times K}(a_1a_2, b_1b_2, \ldots, k_1k_2) = \text{rmax}\{V^{-}_{A}(a_1a_2), V^{-}_{B}(b_1b_2), \ldots, V^{-}_{K}(k_1k_2)\} \leq \text{rmax}\{\text{rmax}\{V^{-}_{A}(a_1), V^{-}_{A}(a_2)\}, \text{rmax}\{V^{-}_{B}(b_1), V^{-}_{B}(b_2)\}, \ldots, \text{rmax}\{V^{-}_{K}(k_1), V^{-}_{K}(k_2)\}\} = \text{rmax}\{\text{rmax}\{V^{-}_{A}(a_1), V^{-}_{B}(b_1), \ldots, V^{-}_{K}(k_1)\}, \text{rmax}\{V^{-}_{A}(a_2), V^{-}_{B}(b_2), \ldots, V^{-}_{K}(k_2)\}\} = \text{rmax}\{V^{-}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{-}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. Therefore $V^{-}_{A\times B\times\ldots\times K}[(a_1, b_1, \ldots, k_1)(a_2, b_2, \ldots, k_2)] \leq \text{rmax}\{V^{-}_{A\times B\times\ldots\times K}(a_1, b_1, \ldots, k_1), V^{-}_{A\times B\times\ldots\times K}(a_2, b_2, \ldots, k_2)\}$. Hence A×B×…×K is a bipolar valued vague subnearring of $R_A\times R_B\times\ldots\times R_K$.

Theorem 2.6. The product of a family of bipolar valued vague subnearrings of nearrings R_i is a bipolar valued vague subnearring of $R_1\times R_2\times\ldots$.

Proof. The proof follows from the Theorem 2.5.

Theorem 2.7. If A = $\langle V^{+}_{A}, V^{-}_{A} \rangle$ is a bipolar valued vague subnearring of nearing R, then $V^{+}_{A}(-u) = V^{+}_{A}(u)$, $V^{-}_{A}(-u) = V^{-}_{A}(u)$, $V^{+}_{A}(u) \leq V^{+}_{A}(o)$ and $V^{-}_{A}(u) \geq V^{-}_{A}(o)$, $\forall$ u∈R, where o is the identity element of (R, +).

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Proof. Let u∈R and o be identity element of (R, +). Now V_A^+(u) = V_A^+(–(–u)) ≥ V_A^+(–u) ≥ V_A^+(u). Therefore V_A^+(u) = V_A^+(–u), ∀ u∈R. And V_A^-(u) = V_A^-(–(–u)) ≤ V_A^-(–u) ≤ V_A^-(u). Thus V_A^-(–u) = V_A^-(u), ∀ u∈R. Also V_A^+(o) = V_A^+(u–u) ≥ rmin{ V_A^+(u), V_A^+(u) }= V_A^+(u). That is V_A^+(o) ≥ V_A^+(u), ∀ u∈R. And V_A^-(o) = V_A^-(u–u) ≤ rmax{ V_A^-(u), V_A^-(u) } = V_A^-(u). Thus V_A^-(o) ≤ V_A^-(u), ∀ u∈R.

Theorem 2.8. Let A = ⟨V_A^+, V_A^-⟩ and B = ⟨V_B^+, V_B^-⟩ be any two bipolar valued vague subsets of the nearrings R and H respectively. Suppose that o and o^l are the identity elements of R and H respectively. If A×B = ⟨ $V_{A\times B}^+$, $V_{A\times B}^-$ ⟩ is a bipolar valued vague subnearring of R×H, then at least one of the following two statements must hold.

(i) V_B^+(o^l) ≥ V_A^+(a) and V_B^-(o^l) ≤ V_A^-(a), ∀ a∈R,

(ii) V_A^+(o) ≥ V_B^+(b) and V_A^-(o) ≤ V_B^-(b), ∀ b∈H.

Proof. By contraposition, suppose that none of the statements (i) and (ii) holds. Then find k∈R and l∈H such that V_A^+(k) > V_B^+(o^l), V_A^-(k) < V_B^-(o^l) and V_B^+(l) > V_A^+(o), V_B^-(l) < V_A^-(o). And $V_{A\times B}^+$(k, l) = rmin{V_A^+(k), V_B^+(l) } > rmin{V_A^+(o), V_B^+(o^l)} = $V_{A\times B}^+$(o, o^l). Also $V_{A\times B}^-$(k, l) = rmax{V_A^-(k), V_B^-(l) } < rmax{ V_A^-(o), V_B^-(o^l)} = $V_{A\times B}^-$(o, o^l). Thus A×B is not a bipolar valued vague subnearring of R×H. Hence either V_B^+(o^l) ≥ V_A^+(a), ∀ a∈R and V_B^-(o^l) ≤ V_A^-(a), ∀ a∈R or V_A^+(o) ≥ V_B^+(b), ∀ b∈H and V_A^-(o) ≤ V_B^-(b), ∀ b∈H.

Theorem 2.9. Let C = ⟨V_C^+, V_C^-⟩ and D = ⟨V_D^+, V_D^-⟩ be any two BVVSNRs of the nearrings R and H, respectively and C×D = ⟨$V_{C\times D}^+$, $V_{C\times D}^-$⟩ be a BVVSNR of R×H. Then the following are true;

(i) if V_C^+(a) ≤ V_D^+(o^l) and V_C^-(a) ≥ V_D^-(o^l), ∀ a∈R, then C is a BVVSNR of R, where o^l is the identity element of H.

(ii) if V_D^+(a) ≤ V_C^+(o) and V_D^-(a) ≥ V_C^-(o), ∀ a∈H, then D is a BVVSNR of H where o is the identity element of R.

(iii) either C is a BVVSNR of R or D is a BVVSNR of H.

Proof. Let a, b∈R. That is (a, o^l), (b, o^l)∈R×H. (i) Using V_C^+(a) ≤ V_D^+(o^l), ∀ a∈R and V_C^-(a) ≥ V_D^-(o^l), ∀ a∈R, then V_C^+(a–b) = rmin{ V_C^+(a–b), V_D^+(o^l+o^l)} = $V_{C\times D}^+$((a–b), (o^l+o^l)) = $V_{C\times D}^+$[(a, o^l)–(b, o^l)] ≥ rmin{$V_{C\times D}^+$(a, o^l), $V_{C\times D}^+$(b, o^l)} = rmin{rmin{V_C^+(a), V_D^+(o^l) }, rmin{V_C^+(b), V_D^+(o^l) }} = rmin{V_C^+(a), V_C^+(b)}. Thus V_C^+(a–b) ≥ rmin{ V_C^+(a), V_C^+(b) }, ∀ a, b∈R. And V_C^+(ab) = rmin{V_C^+(ab), V_D^+(o^{l}o^l) } = $V_{C\times D}^+$((ab), (o^{l}o^l)) = $V_{C\times D}^+$[(a, o^l)(b, o^l)] ≥ rmin {$V_{C\times D}^+$(a, o^l), $V_{C\times D}^+$(b, o^l) } = rmin{ rmin{ V_C^+(a), V_D^+(o^l) }, rmin{ V_C^+(b), V_D^+(o^l) }} = rmin{ V_C^+(a), V_C^+(b) }. Thus V_C^+(ab) ≥ rmin{ V_C^+(a), V_C^+(b) }, ∀ a, b∈R. Also V_C^-(a–b) = rmax{ V_C^-(a–b), V_D^-(o^l+o^l)} = $V_{C\times D}^-$((a–b), (o^l+o^l)) = $V_{C\times D}^-$[(a, o^l)–(b, o^l)] ≤ rmax {$V_{C\times D}^-$(a, o^l),

$V^-_{C\times D}$(b, o^{l})} = rmax{ rmax{ V^-_C(a), V^-_D(o^{l}) }, rmax{V^-_C(b), V^-_D(o^{l}) }} = rmax {V^-_C(a), V^-_C(b)}. Therefore V^-_C(a–b) ≤ rmax{V^-_C(a), V^-_C(b)}, ∀ a, b∈R. And V^-_C(ab) = rmax {V^-_C(ab), V^-_D(o^{l}o^{l}) } = $V^-_{C\times D}$((ab), (o^{l}o^{l})) = $V^-_{C\times D}$[(a, o^{l})(b, o^{l})] ≤ rmax{ $V^-_{C\times D}$(a, o^{l}), $V^-_{C\times D}$(b, o^{l})} = rmax{ rmax{V^-_C(a), V^-_D(o^{l})}, rmax{V^-_C(b), V^-_D(o^{l})}}= rmax{V^-_C(a), V^-_C(b)}. That is V^-_C(ab) ≤ rmax{ V^-_C(a), V^-_C(b)}, ∀ a, b∈R. Hence C is a BVVSNR of R. (ii) Using V^+_D(a) ≤ V^+_C(o), ∀ a∈H and V^-_D(a) ≥ V^-_C(o), ∀ a∈H, then V^+_D(a–b) = rmin{V^+_D(a–b), V^+_C(o+o) } = $V^+_{C\times D}$((o+o), (a–b)) = $V^+_{C\times D}$[(o, a)–(o, b)] ≥ rmin{$V^+_{C\times D}$(o, a), $V^+_{C\times D}$(o, b)} = rmin{ rmin{V^+_C(o), V^+_D(a)}, rmin{ V^+_C(o), V^+_D(b)}} = rmin{V^+_D(a), V^+_D(b)}. Thus V^+_D(a–b) ≥ rmin{V^+_D(a), V^+_D(b)}, ∀ a, b∈H. And V^+_D(ab) = rmin{V^+_D(ab), V^+_C(o.o)} = $V^+_{C\times D}$((o.o), (ab)) = $V^+_{C\times D}$[(o, a)(o, b)] ≥ rmin{$V^+_{C\times D}$(o, a), $V^+_{C\times D}$(o, b)} = rmin{rmin{ V^+_C(o), V^+_D(a)}, rmin{ V^+_C(o), V^+_D(b)}} = rmin{V^+_D(a), V^+_D(b)}. Thus V^+_D(ab) ≥ rmin{V^+_D(a), V^+_D(b)}, ∀ a, b∈H. Also V^-_D(a–b) = rmax{V^-_D(a–b), V^-_C(o+o)} = $V^-_{C\times D}$((o+o), (a–b)) = $V^-_{C\times D}$[(o, u)–(o, b)] ≤ rmax{$V^-_{C\times D}$(o, a), $V^-_{C\times D}$(o, b)} = rmax{ rmax{V^-_C(o), V^-_D(a)}, rmax{V^-_C(o), V^-_D(b)}} = rmax{V^-_D(a), V^-_D(b)}. That is V^-_D(a–b) ≤ rmax{ V^-_D(a), V^-_D(b)}, ∀ a, b∈H. And V^-_D(ab) = rmax{V^-_D(ab), V^-_C(o.o)} = $V^-_{C\times D}$((o.o), (ab)) = $V^-_{C\times D}$[(o, u)(o, b)] ≤ rmax{$V^-_{C\times D}$(o, a), $V^-_{C\times D}$(o, b)}= rmax{rmax {V^-_C(o), V^-_D(a)}, rmax{V^-_C(o), V^-_D(b)}} = rmax{V^-_D(a), V^-_D(b)}. Thus V^-_D(ab) ≤ rmax{V^-_D(a), V^-_D(b)}, ∀ a, b∈H. Hence D is a BVVSNR of H. Hence (iii) is clear.

Theorem 2.10. Let A = ⟨V^+_A, V^-_A⟩ be a BVVSNR of a nearring R. (i) If V^+_A(x–y) = [0], then either V^+_A(x) = [0] or V^+_A(y) = [0] for x, y in R. (ii) If V^+_A(xy) = [0], then either V^+_A(x) = [0] or V^+_A(y) = [0] for x, y in R. (iii) If V^-_A(x–y) = [0], then either V^-_A(x) = [0] or V^-_A(y) = [0] for x, y in R. (iv) If V^-_A(xy) = [0] then either V^-_A(x) = [0] or V^-_A(y) = [0] for x, y in R.

Proof. Let x, y in R. (i) By the definition, V^+_A(x–y) ≥ rmin {V^+_A(x), V^+_A(y)} which implies that [0] ≥ rmin{V^+_A(x), V^+_A(y)}. Therefore either V^+_A(x) = [0] or V^+_A(y) = [0]. (ii) By the definition, V^+_A(xy) ≥ rmin{V^+_A(x), V^+_A(y)} which implies that [0] ≥ rmin{V^+_A(x), V^+_A(y)}. Therefore either V^+_A(x) = [0] or V^+_A(y) = [0]. (iii) By the definition, V^-_A(x–y) ≤ rmax{V^-_A(x), V^-_A(y)} which implies that [0] ≤ rmax{V^-_A(x), V^-_A(y)}. Therefore either V^-_A(x) = [0] or V^-_A(y) = [0]. (iv) By the definition, V^-_A(xy) ≤ rmax{V^-_A(x), V^-_A(y)} which implies that [0] ≤ rmax {V^-_A(x), V^-_A(y) }. Therefore either V^-_A(x) = [0] or V^-_A(y) = [0].

Theorem 2.11. If A = ⟨V^+_A, V^-_A⟩ is a BVVSNR of a nearring R, then H = { x∈R | V^+_A(x) = [1], V^-_A(x) = [−1] } is either empty or a subnearring of R.

Proof. If no element satisfies this condition then H is empty. If x and y in H then V_A^+(x−y) ≥ rmin{ V_A^+(x), V_A^+(y)} = rmin{[1], [1]} = [1]. Therefore V_A^+(x−y) = [1]. And V_A^+(xy) ≥ rmin{ V_A^+(x), V_A^+(y)} = rmin{[1], [1]} = [1]. Therefore V_A^+(xy) = [1]. Also V_A^-(x+y) ≤ rmax{ V_A^-(x), V_A^-(y)} = rmax{[−1], [−1]} = [−1]. Therefore V_A^-(x−y) = [−1]. And V_A^-(xy) ≤ rmax{ V_A^-(x), V_A^-(y)} = rmax{[−1], [−1]} = [−1]. Therefore V_A^-(xy) = [−1]. That is x+y∈H and xy∈H. Hence H is a subnearring of R. Hence H is either empty or a subnearring of R.

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