STUDY OF ANISOTROPIC COSMOLOGICAL MODEL IN A MODIFIED F(R,T) THEORY OF GRAVITY

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Abstract:

Current data indicate that, on a large enough scale, the cosmos is homogenous and isotropic. This does not preclude the possibility that some anisotropy was present early in the evolution of the universe and was subsequently attenuated. In view of this idea, interest in the homogeneous but anisotropic Bianchi models has been raised. Second, modified gravity has attracted a lot of attention recently due to the challenges the traditional ΛCDM model faces in general relativity. As a result, this research examined the Bianchi type-I cosmological model in f(R,T)-modified gravity. Based on certain cosmographic concepts, a particular form of the deceleration parameter was postulated, leading to a model that demonstrated a transition from early deceleration to late-time acceleration.

Keywords: Deceleration parameter; f(R,T) theory; cosmological term; Bianchi type-I universe.

Introduction:

According to recent theoretical and experimental research, the expansion of our universe is speeding at the moment, and "dark energy," an unidentified type of matter, is mostly responsible for this acceleration. Positive energy density combined with negative pressure is one of this dark energy's most intriguing features. According to data from Plank and the Wilkinson Microwave Anisotropy Probe (WMAP), the universe is made up of around 5% baryonic matter, 26.5% dark matter, and 68.5% dark energy. There are two possible ways in which dark energy manifests. The first is by the use of so-called exotic matter, that is, either with respect to the cosmological constant or by applying the equation of state parameter (EOS) $\omega = p/\rho$, where ρ is the energy density and p is the pressure.

The cosmological constant Λ which Albert Einstein used in his field equations to produce a static cosmos is today considered a good dark energy representation to explain the universe's increasing expansion. Nonetheless, it is currently surrounded by cosmological riddles like the cosmic coincidence and fine tuning issues. In the past several years, numerous modified theories of gravity, such as f(R), f(T), f(G), and f(R, T) gravity, have been investigated in an effort to explain the mechanism of the late-time acceleration, dark matter, and dark energy. These theories were proposed to account for dark energy and other cosmological anomalies. Among them, f(R) gravity is noteworthy since it has been thoroughly studied by a number of writers. An arbitrary function of R takes the role of R in the Einstein-Hilbert action in f(R) gravity. A further suggestion for explaining late-time acceleration is the recently created f(T) gravity. This theory represents a generalised form of tele-parallel gravity, where the Levi Civita link is replaced with the Weitzenbock connection. This theory's ability to explain the acceleration of current events without mentioning dark energy is what makes it so intriguing. Introduced by Harko et al., f(R, T) gravity is another modified theory that has garnered a lot of interest recently.

An arbitrary function of the Ricci scalar R and the energy momentum tensor's trace T define the gravitational Lagrangian in this theory. It is observed that the dependence on T can be explained by an imperfect fluid or by quantum processes. Harko and his colleagues examined a few particular variants of the function f(R, T) in their work. This theory may be viewed as a more practical explanation for the universe's acceleration phase. We will now briefly highlight a few more authors who have studied different features of the Bianchi type-I model in f(R, T) gravity. Adhav discovered the locally rotationally symmetric (LRS) Bianchi type-I models. Exponent and power law solutions for the Bianchi type-I model with ideal fluid were examined by Sharif and Zubair. Shamir discovered models with a constant deceleration parameter. By selecting a nonlinear form for the deceleration parameter, Bianchi types I and V bulk viscous solutions were derived by Ram and Kumari. Singh and Bishi looked at a model with a quadratic equation of state and a cosmological constant. By assuming a linearly variable deceleration value, Sahoo and Sivakumar discovered LRS models with a dynamic cosmological parameter.

Sahoo and Reddy discovered bulk viscous LRS models by employing a certain time-dependent deceleration parameter. Yadav used a hybrid expansion law for the scale factor and discovered a transitioning solution. Sharma et al. talked about the LRS models' stability. By selecting two appropriate forms of the scale factor, Pradhan et al. were able to find solutions that showed a shift from early deceleration to late-time acceleration. Bulk viscous LRS models were studied by Yadav et al. by selecting a hybrid form for the scale factor. In this study, we assumed a certain form for the deceleration parameter as a function of the Hubble parameter and analyzed the Bianchi type-I cosmological model.

Modified $f(R,T)$ Theory of Gravity:

The action of $f(R,T)$ gravity is given by:

$$S = \int \sqrt{g}\left(\frac{-1}{16\pi G} f(R,T) + L_m\right) d^4x \tag{1}$$

where g is the determinant of the metric tensor g_{ij}; $f(R, T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor T_{ij} i.e., $T = g^{ij} T_{ij}$; and L_m is the matter Lagrangian density. It is important to note that the f (R, T) theory of gravity is an expansion of the f (R) theory and a modification of general relativity. Similar to f (R) gravity models, the field equations are generated by varying the combined action of the field and matter and equating this variation to zero. Now, using gravitational units ($8\pi G = 1$, $c = 1$) and varying the action S in (1) with respect to the metric tensor g_{ij}, we obtain the field equations in $f(R, T)$ gravity as:

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$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij} - \nabla_i\nabla_j)f_R(R,T)$$
$$= 8\pi T_{ij} - f_r(R,T)T_{ij} - f_r(R,T)\theta_{ij} \quad (2)$$

Where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_R(R,T) = \frac{\partial f(R,T)}{\partial T}$, R_{ij} is the Ricci tensor and T_{ij} is the energy momentum tensor given by :

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m}{\delta g^{ij}} \quad (3)$$

In equation (2) $\nabla_i\nabla_j$ is the D' Alembertian operator, where ∇_i represents the covariant derivative and:

$$\theta_{ij} = -2\text{Tij} + g_{ij}L_m - 2g^{\mu v}\frac{\partial^2 L_m}{\partial g^{\mu v} g^{ij}}, \quad (4)$$

Equation (2) produces an important relation upon contraction that links the energy-momentum tensor's trace T and the Ricci scalar R:

$$f_R(R,T)R + 3f_R(R,T) - 2f(R,T) = (8\pi - f_R(R,T)T - f_R(R,T)\theta, \quad (5)$$

where $\theta = \theta_j^i$ If we assume that the matter Lagrangian density L_m depends only on the metric tensor component g_{ij} rather than its derivatives, then Equation (3) is reduced to the form:

$$\text{Tij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \quad (6)$$

The matter's energy-momentum tensor has the following shape for a perfect fluid distribution:

$$\text{Tij} = (\rho + \text{p})u_i u_j - \text{p}g_{ij}, \quad (7)$$

where ρ and p are the energy density and pressure of the fluid, respectively. Here u^i is the four-velocity vector satisfying $u^i u_i = -1$ and $u^i \nabla_j u_i = 0$. Now, using the fact that $L_m = -p$, Equation (4) can be rewritten as:

$$\theta_{ij} = -2\text{Tij} - \text{p}g_{ij} \quad (8)$$

Due to this, the field Equation (2) has the following form:

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij} - \nabla_i\nabla_j)f_R(R,T)$$
$$= 8\pi T_{ij} - f_r(R,T)T_{ij} - f_r(R,T)\theta_{ij} \quad (9)$$

Harko et al. [12] have considered three possible forms of the function $f(R, T)$:

$$\text{f(R,T)} = \begin{cases} \text{R} + 2f_1(\text{T}) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(T)f_3(T) \end{cases} \quad (10)$$

In the present study, we shall concentrate on the first form of $f(R, T)$ i.e., $f(R, T) = R + 2f_1(T)$— and choose $f_1(T) = \lambda T$, where λ is an arbitrary constant. For this considers- ation and energy-momentum tensor (7), Equation (9) is reduced to the form:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(1 + 2\lambda)T_{ij} + \lambda(T + 2p)g_{ij} \quad (11)$$

Now, the cosmological term in Einstein's field equations can be expressed as:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \Lambda g_{ij} \quad (12)$$

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By comparing Equations (11) and (12), and by taking the parameter λ to be small, we can make the identification $\Lambda = \Lambda(T) = \lambda(T + 2p)$. Therefore, in the $f(R,T)$ the- ory of gravity, the field equations with a variable cosmological parameter Λ(T) can be expressed as:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -(1 + 2\lambda)T_{ij} + \Lambda g_{ij} \tag{13}$$

In the case of a perfect fluid, the trace T of the energy-momentum tensor can be written as $T = \rho - 3p$. The cosmological parameter can be written as:

$$\Lambda = \lambda(\rho - p) \tag{14}$$

It can clearly be seen from Equation (13), which follows from Equation (11), that the usual energy conservation law does not hold in general in the $f(R, T)$ theory. Shabani and Zaiae have noted that, from a thermodynamic perspective, the non-conservation of energy suggests an irreversible process of matter production. It is anticipated that basic particle physics will be able to support this procedure. Energy transfer from the gravitational field to the formed matter particles is correlated with such particle formation. In a different work, the same writers examined the effects of energy conservation. They discovered that late-time stable accelerating solutions are not a general characteristic in f(R, T) gravity if energy conservation holds true. On the other hand, a wide class of stable solutions with a dynamic λ(T) and late-time acceleration can be found with energy non-conservation.

Let's start by determining whether or not we have energy conservation in our situation. Equation (13) exhibits zero divergence on the LHS due to the Bianchi identities. This suggests that there can be no divergence on the RHS either. This indicates that the standard energy conservation law is applicable in a single scenario. This occurs when $\partial\rho/\partial t = \partial p/\partial t$; otherwise, energy is generally not conserved, as this study demonstrates.

Field Equations

The following line element represents the gravitational field for a spatially homogenous and anisotropic Bianchi type-I space-time:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2. \tag{15}$$

Where A, B, C are metric functions of the cosmic time t. For the Bianchi type-I space-time (14), the field Equation (13) in $f(R,T)$ gravity yields the following dynamical equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = \Lambda - (1 + 2\lambda)p \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = \Lambda - (1 + 2\lambda)p \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = \Lambda - (1 + 2\lambda)p \tag{18}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = \Lambda - (1 + 2\lambda)p \tag{19}$$

where the ordinary derivative with respect to cosmic time t is represented by an over dot. The matter content is assumed to follow the standard equation of state:

$$p = \omega\rho, \qquad -1 \le \omega \le 1. \tag{20}$$

For the Bianchi type-I space-time, the spatial volume (V) and average scale factor (a) are given by, respectively:

$$V=ABC, \tag{21}$$

$$a=(ABC)^{\frac{1}{3}}=V^{\frac{1}{3}} \tag{22}$$

An average Hubble parameter (H) for the Bianchi type-I is defined by:

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{23}$$

Where $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}$ and $H_3=\frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y and z axis respectively.

Equations (22) and (23) can also be written in the form:

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{24}$$

The expansion scalar(θ), shear scalar(σ), and anisotropy parameter A_m are defined as, respectively:

$$\theta=3\mathrm{H}=3\frac{\dot{\mathrm{a}}}{\mathrm{a}} \tag{25}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-\frac{1}{3}\theta^2\right), \tag{26}$$

$$A_m=\frac{2\sigma^2}{3H^2} \tag{27}$$

From eq. (16)-(18) we can obtain the following equations

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}\right)=0 \tag{28}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}\left(\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}\right)=0, \tag{29}$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\frac{\ddot{B}}{B}\left(\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}\right)=0, \tag{30}$$

Then the equations:

$$\frac{A}{B}=c_1exp\left(d_1\int\frac{dt}{a^3}\right), \tag{31}$$

$$\frac{B}{C}=c_2exp\left(d_2\int\frac{dt}{a^3}\right), \tag{32}$$

$$\frac{A}{C}=c_3exp\left(d_3\int\frac{dt}{a^3}\right), \tag{33}$$

Where c_1, c_2, c_3 and d_1, d_2, d_3 are the constant of integration.

The deceleration parameter (q) is defined as:

$$q=-\frac{a\ddot{a}}{\dot{a}^2} \tag{34}$$

From equation (13)-(16) can be expressed in terms of H, q and σ as:

$$3H^2-\sigma^2=\Lambda+(1+2\lambda)\rho, \tag{35}$$

$$H^2(2q-1)-\sigma^2=(1+2\lambda)\rho-\Lambda \tag{36}$$

Equations (14) and (16)–(20), which are obtained from the field Equation (13), represent six equations in the six unknown quantities—i.e., A, B, C, ρ, p, and Λ, respectively. Hence, one can try to solve for the system directly. But this is really challenging. Furthermore, we are interested in finding appropriate cosmological solutions that show an early-to-late acceleration transition, consistent with current data. We assume for the purposes of this inquiry that the deceleration parameter q is

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expandable in relation to the Hubble parameter H. A multitude of assumptions can be employed in order to solve this system. We take into account the time-dependent deceleration parameter q since observations show that the cosmos is undergoing a phase transition from the previous decelerating expansion to the current accelerating one.

The deceleration parameter is a geometric quantity that, depending on its sign, characterises the universe's acceleration or slowdown. Within this framework, it is established that the universe expands at an accelerating rate if q < 0; it expands at a decelerating rate if q > 0; it expands at a constant rate if q = 0; and the accelerating expansion is known as super-exponential expansion if q < 1.

Inspired by the aforementioned, we choose the deceleration parameter as a function of the Hubble parameter H, as suggested by Tiwari et al., to explain the behaviour of the universe:

$$q = \alpha - \frac{\beta}{H}. \tag{37}$$

Here, α and β are constants and $\beta > 0$. This form of the deceleration parameter yields the required transition from positive to negative as we desire. Equation (40) leads to the following solution for the scale factor:

$$a = k_1\left(e^{\beta t} - 1\right)^{\frac{1}{1+\alpha}}, \tag{38}$$

Where k_1 is a constant.

The form of the generalized mean Hubble parameter H

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{39}$$

Where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions.

The spatial volume V, Hubble parameter H, expression scalar θ, shear scalar σ^2 , anisotropy parameter A_m, and declaration parameter q take the following forms, respectively:

$$V = k_1^3\left(e^{\beta t} - 1\right)^{\frac{1}{1+\alpha}}, \tag{40}$$

$$H = \frac{\beta e^{\beta t}}{(1+\alpha)(e^{\beta t}-1)}, \tag{41}$$

$$\theta = \frac{3\beta}{(1+\alpha)\left(1-e^{-\beta t}\right)}, \tag{42}$$

$$\sigma^2 = \frac{k_1^2+k_2^2+k_1k_2}{3k_1^6\left(e^{\beta t}-1\right)^{\frac{6}{1+\alpha}}}, \tag{43}$$

$$A_m = \frac{2\left(k_1^2+k_2^2+k_1k_2\right)(1+\alpha)^2}{9\beta^2k_1^6e^{2\beta t}\left(e^{\beta t}-1\right)^{\frac{4-2\alpha}{1+\alpha}}} \tag{44}$$

Equations (35)–(45) are determined essentially from (44), and are the kinematic quantities. The field Equation, on the other hand is basically used to determine the dynamical quantities, the energy density ρ, pressure p, and cosmological parameter Λ.

From Equations (17)–(19), we obtain energy density ρ and pressure p:

$$\rho = \frac{1}{(1+\omega)(1+2\lambda)}\left[\frac{2\beta^2 e^{\beta t}}{(1+\alpha)\left(e^{-\beta t}-1\right)^2} - \frac{2\left(k_1^2+k_2^2+k_1k_2\right)}{3k_1^6\left(e^{\beta t}-1\right)^{\frac{6}{1+\alpha}}}\right], \tag{45}$$

$$p = \frac{\omega}{(1+\omega)(1+2\lambda)}\left[\frac{2\beta^2 e^{\beta t}}{(1+\alpha)\left(e^{-\beta t}-1\right)^2} - \frac{2\left(k_1^2+k_2^2+k_1k_2\right)}{3k_1^6\left(e^{\beta t}-1\right)^{\frac{6}{1+\alpha}}}\right] \tag{46}$$

The cosmological parameter $\Lambda = \lambda(\rho - p)$ is given by:

$$\Lambda = \lambda\left[\frac{2(1-\omega)\beta^2 e^{\beta t}}{(1+\omega)(1+\alpha)(1+2\lambda)\left(e^{-\beta t}-1\right)^2} - \frac{(1+\omega)}{(1+\omega)(1+2\lambda)}\frac{2\left(k_1^2+k_2^2+k_1k_2\right)}{3k_1^6\left(e^{\beta t}-1\right)^{\frac{6}{1+\alpha}}}\right] \tag{47}$$

For our Bianchi model (14), we observe that the spatial volume V is zero and expansion scalar θ are infinite at $t = 0$. Thus, the universe starts evolving with zero volume and an infinite rate of expansion at $t = 0$. Equations (34)–(36) and (41) show that the scale factors also vanish at $t = 0$, hence the model has a "point type" singularity at the initial epoch. Initially, at $t = 0$ the Hubble parameter H and shear scalar σ^2 are infinite. The energy density ρ, pressure p and cosmological constant Λ are also infinite. As t tends to infinity, V becomes infinitely large, whereas σ^2 approaches zero. Later, the energy density ρ and pressure p converge to zero. The cosmological parameter Λ also approaches a constant later. The deceleration parameter q for the model is a constant α at $t = 0$, and as t increases—i.e., when it is $(1/\beta)\, log(1 + \alpha)$— q is zero, which shows that there will be a transition to acceleration. It is equal to 1 when t tends to infinity, which shows that the model describes the accelerating phase of the universe. The anisotropy parameter A_m gives a measure of the anisotropy of the model, and is given by Equation (44), which is large early on as $t0$ but decreases very rapidly.

As a matter of interest, the solution for $\Lambda = 0$, which also means $\lambda = 0$ from Equation (14), can now be easily given. All the kinematic quantities are the same as before, viz., Equations (41)–(50). The density and pressure are given by:

$$\rho = p = \frac{1}{(1+\omega)}\left[\frac{2\beta^2 e^{\beta t}}{(1+\alpha)\left(e^{-\beta t}-1\right)^2} - \frac{2\left(k_1^2+k_2^2+k_1k_2\right)}{3k_1^6\left(e^{\beta t}-1\right)^{\frac{6}{1+\alpha}}}\right] \tag{48}$$

LRS Bianchi Type –II cosmological model with constant declaration parameter. Which shows that this model is an accelerating model of the universe. From eq. (29), we conclude that, the model has an initial singularity at $t = 0$ and expands with time. The line element with scale factors $A(t)$ and $B(t)$, given by equations (30) and (31) gives an exact stiff fluid solution of the universe . scale factors A(t) vanishes while the other one $B(t)$ diverges at large time. From the above results we can discuss the physical behaviour of the universe. From eq. (32), (35)-(38),we observe that, at initial epoch energy density(ρ), the Hubble parameter (H), expansion scalar (θ), the share scalar (σ), the anisotropy parameter (Δ) all diverges and vanishes at large value of time. since, that $\frac{\sigma^2}{\theta}$ does not tends to zero at $t \rightarrow \infty$, which indicates that this model of universe does not approaches isotropy at late times. positive constant value of declaration parameter shows that the model has a decelerating expansion $(q > 0)$.

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Conclusions:

Within the context of f(R,T) gravity theory, we examined the homogeneous and anisotropic Bianchi Type-I & II model in this work. To precisely solve the field equations, take (R,T) =R+2f(T), where f(T) =λt. The article includes examination of important cosmological parameters for the two alternative theories, assuming a constant jerk value (j=1). Power law comes first, followed by exponential law. Model I applies to the first epoch and does not reach isotropy in later times in the cosmos. Decelerating expansion is indicated by a positive constant value of the declaration parameter (q>0). Additionally, model II corresponds to fast expansion and is valid for late times. We have examined and talked about the different kinematic and physical factors. Because the flat ΛCDM model has a jerk parameter (j=1), our work's models are predicated on it.

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